Akademia Górniczo-Hutnicza
Mechanical Engineering Degree

AGH

AKADEMIA GÓRNICZO-HUTNICZA
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Individual Research Project

WIND TURBINE ROTOR MODEL WITH A HORIZONTAL AXIS

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1. Introduction

Nowadays, we are already beginning to see the consequences of the uncontrolled use of non-renewable energies. The poles are melting, sea levels are rising, ecosystems are changing, etc., and life on this planet may not be as we know it in the future. Moreover, this process of increasing the Earth's average temperature is irreversible, so the only thing we can do is to prevent this process from going any further.

The different types of renewable energies are wind energy, solar energy, hydraulic energy, solar thermal energy, biomass energy, geothermal energy and many more types that are under investigation.

In this project I am going to analyse and simulate a wind turbine, so it is necessary to analyse the current situation of wind energy, looking at its background, technologies and its vision for the future.

1.1 Aim of this project

The aim of this project is to determine and design the ability of horizontal wind turbine parameters and how it will work on a wind tunnel with the dimensions of 1,3x1,3m.

Picture 1. AGH Wind tunnel

The objective of this project are:

- Make and compile some literature and information about wind turbines, specifically on wind turbines with horizontal axis.
- Determine the best propeller design for the wind turbine, attempting to obtain the maximum power from the wind with the maximum efficiency.
- Numerical testing of the airfoil designed before in static conditions. Simulations to be carried in CFD package ANSYS Fluent19.1.
- According to the design, make some numerical calculations about the power and the efficiency of our turbine and then compare them to the prototype on the wind tunnel.
- Finally, Determine sources of error and propose further techniques to improve the experimental testing.

Out of these objectives,the research questions to approach are:

- How a horizontal axis wind turbine works and what are the advantages and disadvantages of these types of wind turbines?
- What are the different powers that we can obtain from a turbine depending on the efficiency?
- How can we use the turbulence models in CFD and how to calculate it?

2.Literature research

2.1 Historical Background

A windmill is a machine that transforms wind into usable energy, which comes from the action of the force of the wind on oblique blades attached to a common shaft. The rotating shaft can be connected to various types of machinery to grind grain, pump water or generate electricity. When the shaft is connected to a load, such as a pump, it is called a windmill. If it is used to produce electricity, it is called a wind turbine generator. Windmills have a remote origin.

The oldest known reference is to a windmill that was used to power an organ in the 1st century common era. The first practical windmills were built in Sistan, Afghanistan, in the 7th century. These were vertical-axis mills with rectangular blades. Apparatuses made of 6 to 8 windmill sails covered with cloth were used to grind grain or draw water.

In Europe the first mills appeared in the 12th century in France and England and spread throughout the continent. They were wooden structures, known as mill towers, which were turned by hand around a central pole to lift their blades into the wind. The tower mill developed in France during the 14th century. It consisted of a stone tower topped by a rotating wooden structure that supported the mill shaft and the mill's upper machinery.

These early examples had a number of common features. A horizontal shaft protruded from the top of the mill. Four to eight blades, with a length of between 3 and 9 metres, came from this shaft. The wooden beams were covered with cloth or wooden planks. The energy generated by the turning of the shaft was transmitted, through a system of gears, to the mill machinery located at the base of the structure. Horizontal shaft mills were widely used in Western Europe to grind wheat from 1180 onwards. One need only recall the now famous windmills in the adventures of Don Quixote. Windmills of this kind still exist, for example, in Holland.

Picture 2. Wind-powered grain mill (*Agaten/Getty Images)*

In the United States, the development of pumping mills, recognisable by their multiple metal sails, was a major factor in enabling agriculture and ranching in vast areas of North America otherwise impossible without ready access to water. These mills contributed to the expansion of railways around the world, supplying the water needs of steam locomotives.

Picture 3. Wind-driven water pump
(https://www.alternative-energy-tutorials.com/wind-energy/)

Modern turbines were developed in the early 1980s, although designs are still under development. The wind energy industry in modern times began in 1979 with the mass production of wind turbines by the manufacturers Kuriant, Vestas, Nordtank, and Bonus.

Those turbines were small by today's standards, with capacities of 20 to 30 kW each. Since then, the size of the turbines has grown enormously, and production has expanded to many countries.

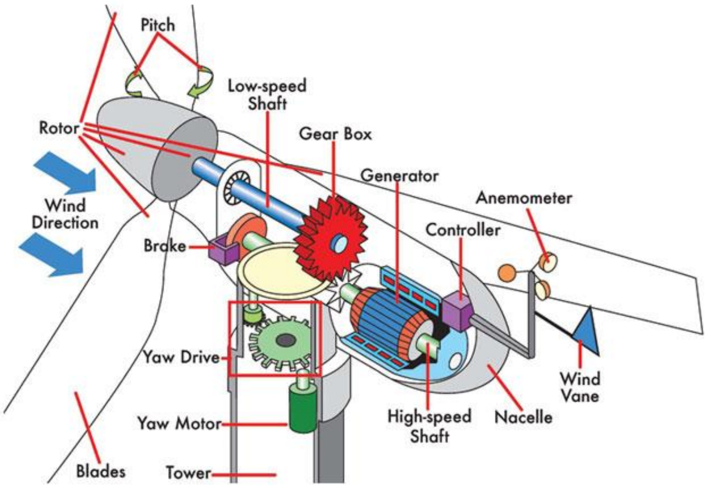
Picture 4. Electric wind turbine (https://futuroelectrico.com/energia-eolica-que-es/)

2.2 Basic idea

A wind turbine is a machine capable of receiving the kinetic energy brought by the wind and converting it into a rotary motion. This rotary motion is used to rotate a shaft and this shaft drives the electrical machine that generates the electrical energy to be fed into the grid. There are two types of wind turbines; with either horizontal or vertical axes. On this project, I will analyse a wind turbine with horizontal axis.

2.2.1 Components of a wind turbine with horizontal axis

The different parts that make up a wind turbine with horizontal axis are:



Picture 5. Wind turbine parts (Some Industrial Systems - Scientific Figure on ResearchGate. Available from: https://www.researchgate.net/figure/Parts-inside-the-wind-turbine_fig5_290094650 [accessed 3 Jun, 2021])

- **Tower:** The tower can be defined as the support element, which, in addition to supporting the nacelle, allows the blades to be at a certain height. The tower is therefore the structural element, normally made of steel, which supports the entire weight of the wind turbine, and is hollow to allow access to the nacelle by the operators.
- **Nacelle:** The nacelle can be described as the engine room of the wind turbine.
  Inside the nacelle there are various mechanisms such as: the gearbox, the main shaft, the regulator or control system, the generator, the brakes and the turning mechanisms.
- **Gearbox:** The function performed by the gearbox is to adjust the rotational speed of the wind turbine shaft to the required
- **Generator:** The generator is one of the main components, which due to the high rotational speed obtained from the gearbox generates electricity.
- **Control system or regulator:** The control system is made up of a set of elements whose function is to check the correct operation of the wind turbine. The control system consists of anemometers, wind vanes, cooling units, power control systems.
  control systems.
- **Brakes:** The braking system allows the mill to stop in case of emergencies or maintenance.
- **Rotor:** The rotor is the element that is attached to the main shaft for the transmission of rotation.
- **Blades:** The blades receive the force of the wind, which is why they are usually manufactured with materials with high resistance to fatigue as they must withstand an average life of 25 years. The most common wind turbines are three-bladed, although there are also single-bladed and two-bladed wind turbines.

2.2.2 Materials and dimensions

There are a lot of requirements that the blades must to meet for its correct operation:

1. The blades need **adequate structural strength** to support the workings conditions.
2. **Fatigue resistance** (in particular to alternating stresses due to vibration).
3. **Stiffness**
4. **Low weight**
5. **Resistance to environmental agents** (erosion, corrosion)

Due to these requirements, there are different materials that we need to keep in mind to the production of the blades. The most commonly used materials are:

- **Steel and aluminium alloys**: which have weight and metal fatigue problems, are currently used only in very small wind turbines.
- **Fibreglass reinforced with polyester resin**, for most modern rotor blades of large wind turbines.
- **Glass fibre reinforced with epoxy resin** ("GRP"). Lighter blades, greater flexibility, less deformation under extreme temperatures, excellent resistance to water absorption.
- **Carbon fibre or aramid** (Kevlar 29 or Kevlar 49) as reinforcing material in strips due to its good mechanical properties, high specific strength, very light blades. Normally these blades are uneconomical for large wind turbines.
- **Glass fibre/carbon fibre composites**.

Firstly, to understand the way that a wind turbine works, we need to know some basic concepts about energy.

2.3 Kinetic energy

Kinetic energy is the energy that objects possess due to their motion.

$KE = \frac{1}{2}mv^2$

$m = mass\ (Kg)$

$v = velocity\ (m/s)$

$KE = Kinetic\ energy\ (J)$

To apply this concept to our project, we also need to know that the air has different density depending on the temperature. It is represented on the following table:

Temperature (^{o}C)	Density (Kg/m^3)
0	1.29
20	1.185
50	1.09
100	0.946

:*Table 1. Density of air*

As we can see on the equation, the velocity of the air has an important weight on our project, furthermore, it is important to know that depending on the location of our wind turbine, the wind speed changes because of the local climate. Due to Saffir-Simpson Scale, we can make the following groups depending on the wind speed:

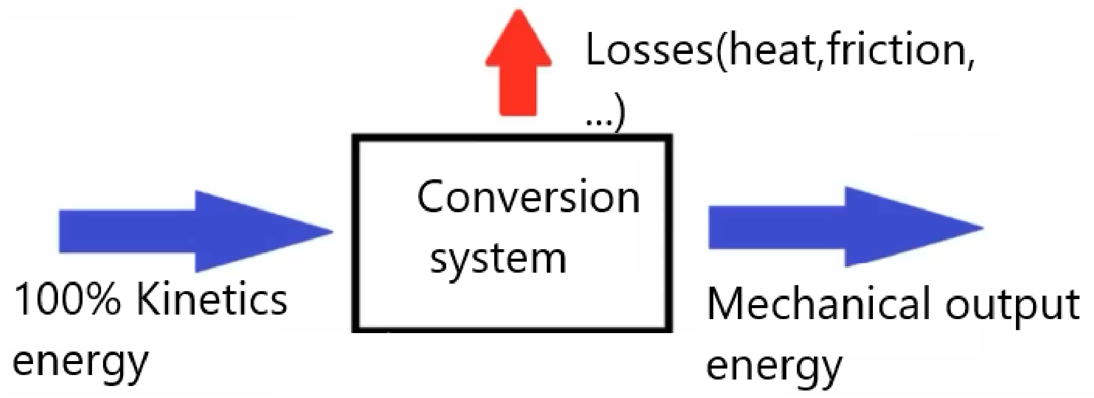
Saffir-Simpson Scale	
Velocity	Summary
0-17m/s(0-62 km/h)	Light to heavy rains and tropical depression
18-32 m/s(63-117km/h)	Tropical storms
33-42m/s(119-153km/h)	Category 1 hurricane
43–49 m/s(154-177km/h)	Category 2 hurricane
50-58m/s(178-208km/h)	Category 3 hurricane
58-70m/s(209-251km/h)	Category 4 hurricane
>70m/s(>252km/h)	Category 5 hurricane

Table 2. Saffir-Simpson Scale

When applying the kinetic energy equation, we obtain the result in joules, however, when the air current is constant, the result is obtained in joules per second. It is important to know that one joule per second is equal to one watt.

2.4 Efficiency

Efficiency is another concept that we must keep in mind because when converting the kinetic energy obtained from the wind into mechanical or electrical energy, losses are produced. These losses can be caused by heat, friction,...



100% efficiency cannot be achieved in wind systems.

To extract energy from the wind, we can only do it by reducing its speed with mechanical systems, we cannot totally stop the wind.In order to extract kinetic energy, a flow is required.

2.5 Drive systems

In wind generators that produce energy by drag energy, we must take into account the following factors:

From the air

P-Pressure
V-velocity
p-Density

From the wind turbine

A-Area
F-Force
Cd and Cl- aerodynamic coefficients

2.5.1 Pressure

Pressure is a physical quantity that measures the projection of force in a perpendicular direction per unit area.

$$P = \frac{F}{A},$$

Where F is the force and A is the Area

There are two kinds of pressure: dynamic pressure and static pressure

- **Static pressure** is the pressure of a fluid, independent of the fluid velocity.

- The **dynamic pressure** depends on the velocity and density of the fluid.

The force exerted by the fluid pressure due to its velocity is known as the **drag force**. To obtain the drag force we need to use the following formula:

$$Fd = \frac{1}{2} \rho v^{2} CdA$$

Where Fd is the drag force, ρ is the density of the fluid, v is the speed of the object relative to the fluid, A is the cross sectional area and Cd is the drag coefficient.

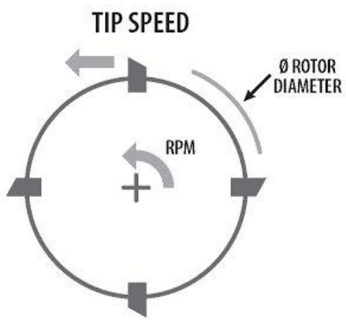
2.5.2 Tip Speed Ratio

All wind systems have a factor called the tip speed ratio. It indicates the ability to convert the wind speed to an amount of rpm of the wind system. This tip speed is calculated by equating the air speed with the tangential speed of the wind system.

To obtain the frequency of our system, we must apply the following equation. It will give us the result in revolutions per second.

$$F = \lambda \frac{V}{2\pi R}$$

Where F is the frequency, V the velocity of the wind, R the radius and λ is the TSR



Picture 6. Tip Speed ratio

The tip speed ratio is the ability of the system to transfer air speed to the wind system. It varies from 0 to 1 in trailing wind systems.

2.5.3 Mechanical parameters

Force is a mechanical quantity that can modify the amount of movement or the shape of materials. Force is measured in Newtons or Kilogram force.

Torque is a force applied to a lever to rotate an object. Torque is measured in units of force and length. The torque is calculated:

$$\Gamma = F\, x\, R$$

Where Γ is the torque, F is the force and R is the radius

2.5.4. Lift coefficient

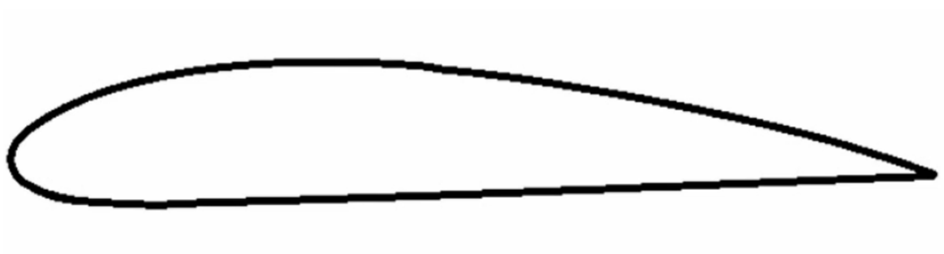
Determines the portion of the resultant force on the propeller that becomes the lift force, which acts on the propeller to cause it to move in the direction of rotation.

2.5.5. Drag coefficient

It determines the portion of the resultant force on the propeller that is converted into drag force, this force is a loss agent so it is important to know it in the design of the propeller. It is also the force that produces bending in the propellers.

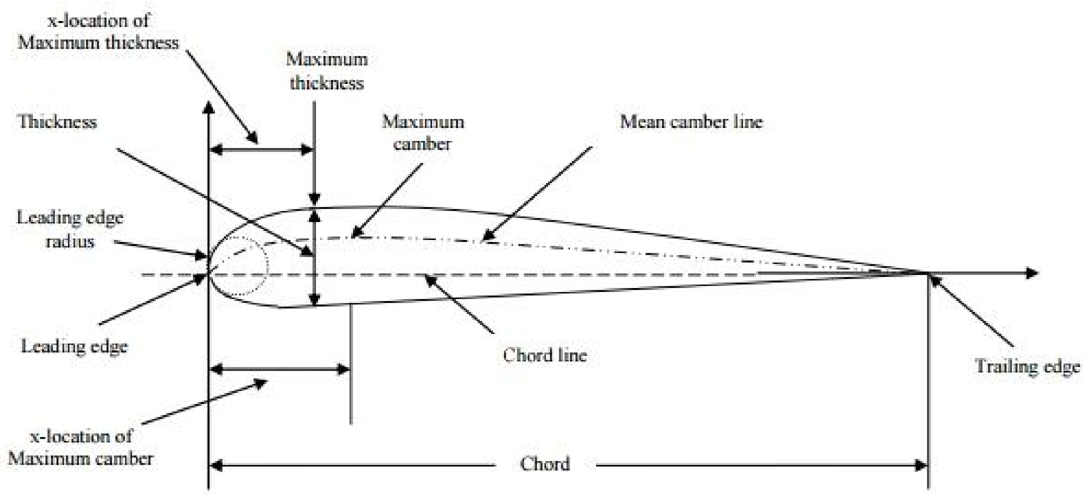
2.5.6. Aerodynamic profiles

It is an object that when moving in a fluid generates a certain distribution of pressures that cause its lift or aerodynamic forces.

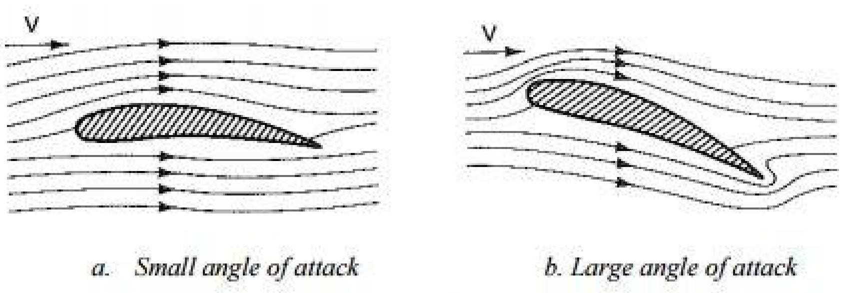


Picture 7. Aerodynamic profile

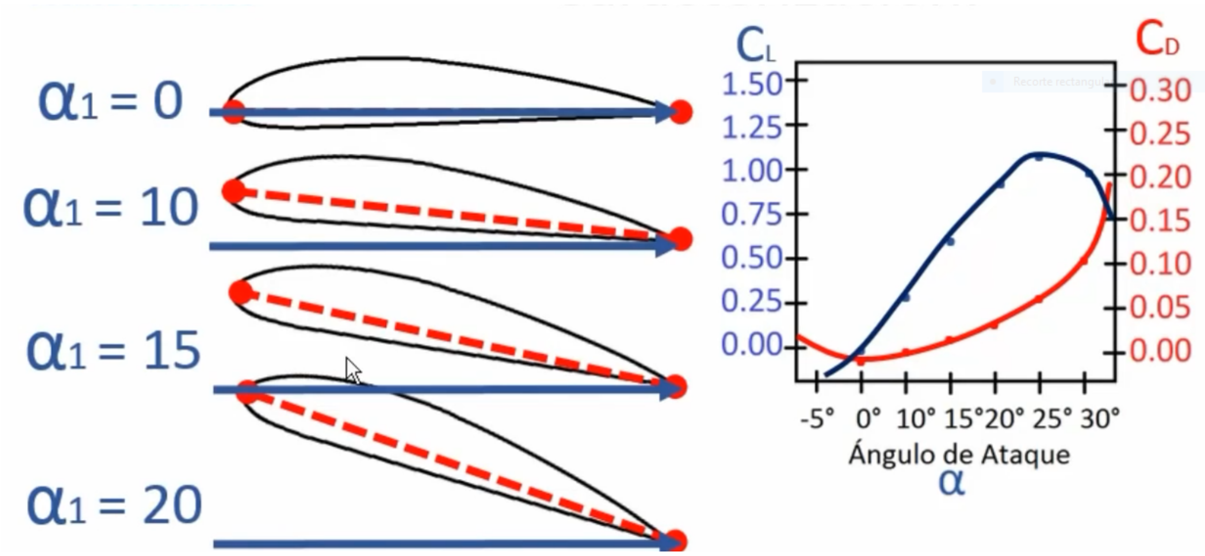
Airfoils are used in aircraft wings, helicopter blades, fans and obviously wind turbines.



Picture 8. Airfoil geometric parameters (pic from wikipedia.org)



Picture 9. Flow around an airfoil (https://optimization.mccormick.northwestern.edu/index.php/Wing_Shape_Optimization)



Picture 10. Airfoil angles of attack

$$Fd = \frac{1}{2}\rho v^2 CdA$$

Where ρ is the density, v the velocity, Cd the drag coefficient and A is the area.

3. Propeller design

3.1. Betz's Law

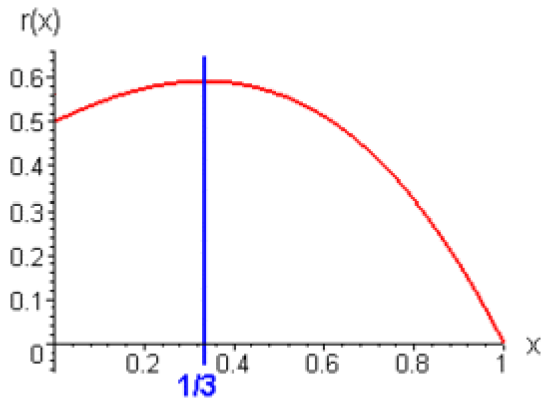
Betz's theorem states that only less than 16/27 (59 %) of the kinetic energy can be converted into mechanical energy using a wind turbine.

$$\dot{m} = \rho A \frac{(v1+v2)}{2}$$

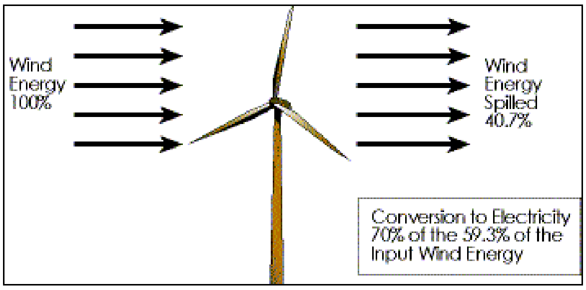
$$P = \frac{1}{2}\dot{m}(v1^2 - v2^2)$$

$$\frac{P}{P0} = \frac{1}{2}(1 - (\frac{v2}{v1})^2)(1 + (\frac{v2}{v1}))$$

Where ṁ is the fluid flow, p the density, A the area, v1 and v2 the velocity through the turbine and P the pressure.



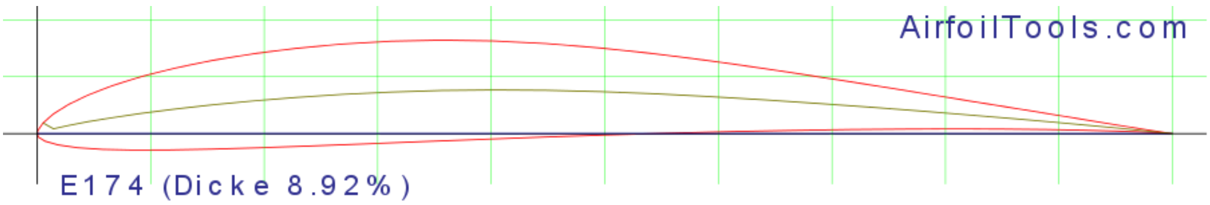
Graph 1. Maximum power (Graphique illustrant la démonstration de la limite de Betz Auteur : Benoit Chauvet @2005)



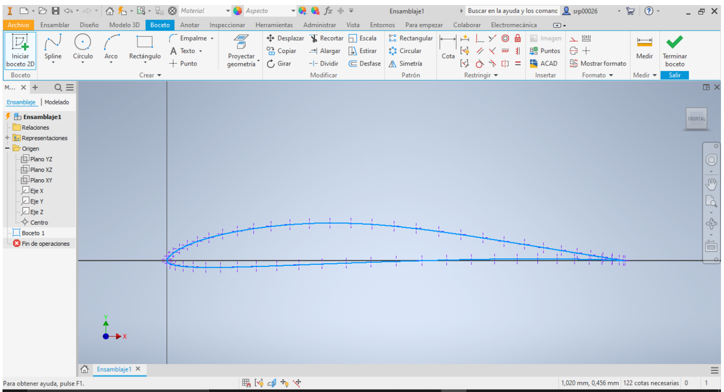
Picture 11. Wind energy conversion

On the following pages, you will find the different steps that I used to design the propeller for our wind turbine. First of all, I'm going to select an aerodynamic profile from the website "http://airfoiltools.com/" which is a database of different aerodynamic profiles depending on the kind of project and kind of wind turbine that we are going to build.

I decided to choose the following aerodynamic profile called *'E174 (Dicke 8.92%) - Eppler E174 low Reynolds number airfoil'* which is perfect for low-power wind turbines, as in the case of the wind turbine to be designed.



Picture 12. Airfoil selected (airfoiltools.com)

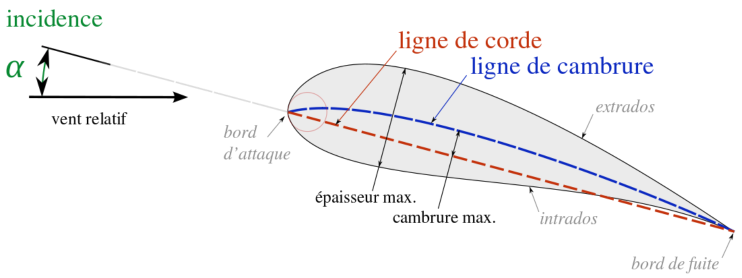


Picture 13. Inventor representation (Inventor Software)

It is a perfect design for our wind turbine because we are going to test it with low wind speeds and tend not to generate boundary layer separation on the profile surface as the fluid traverses it, maintaining laminar flow.

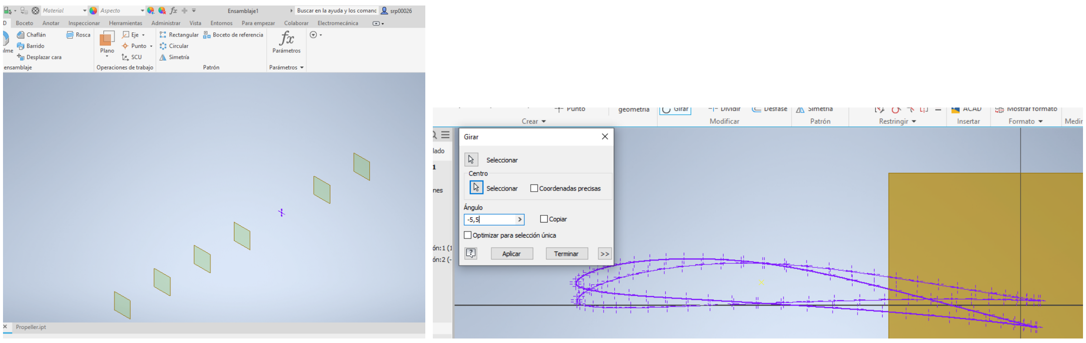
For an airfoil designed under aerodynamic principles, in a wind turbine, two zones of fluid circulation on its surfaces are defined:

- Extrados: this is the region of fluid circulation over the airfoil where the air layers are in depression.
- Intrados: this is the region of fluid circulation over the airfoil where the air layers are under overpressure.



Picture 14. Airfoil diagram (Olivier Cleynen Wikipedia)

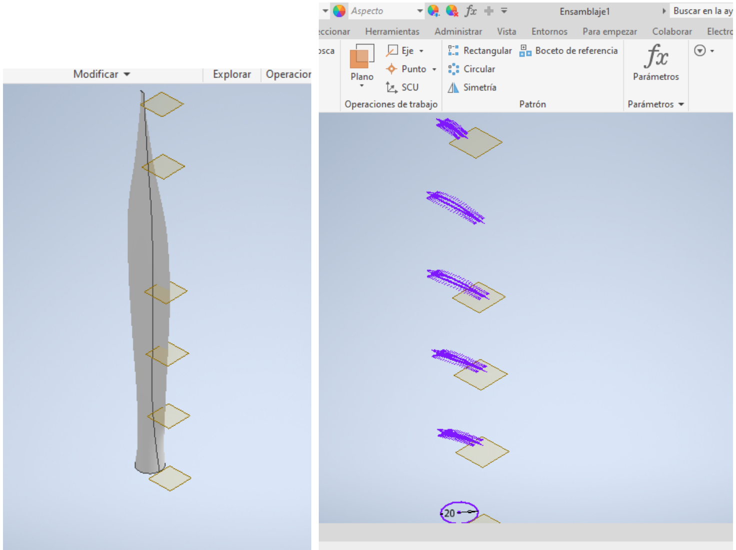
After choosing the propeller design, I make the design with the software INVENTOR



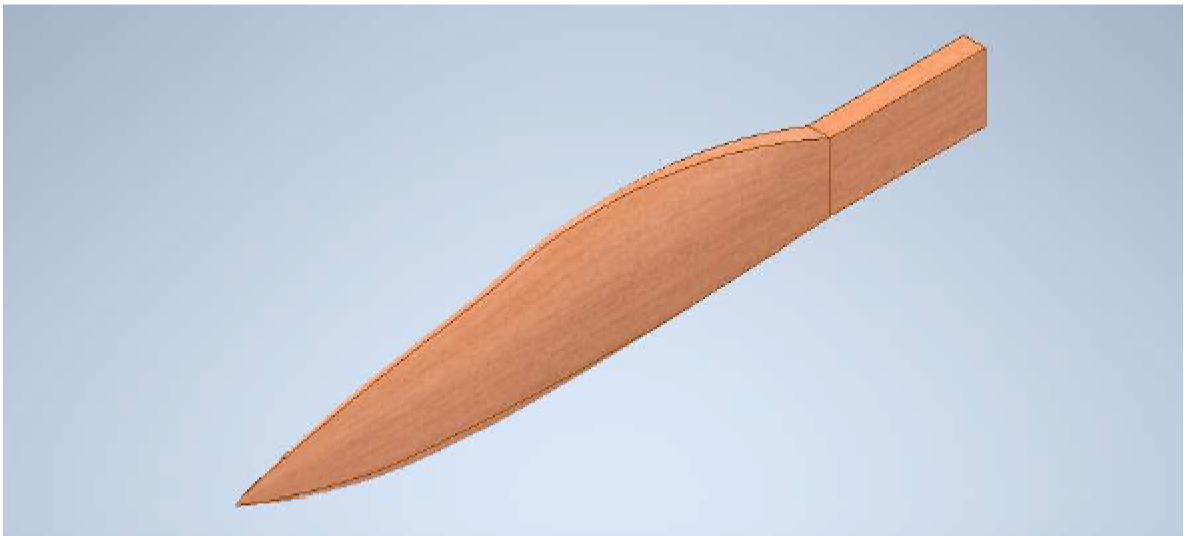
Picture 15. Inventor progress (Inventor Software)

Plane	Scale	Grades
1	0,6	-5,5º
2	0,7	0
3	1	5,5
4	0,5	11
5	0,1	16,5

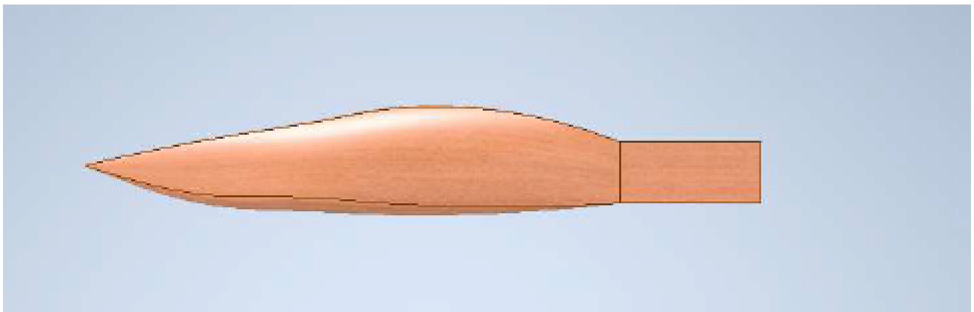
Table 3. Inventor planes and angles



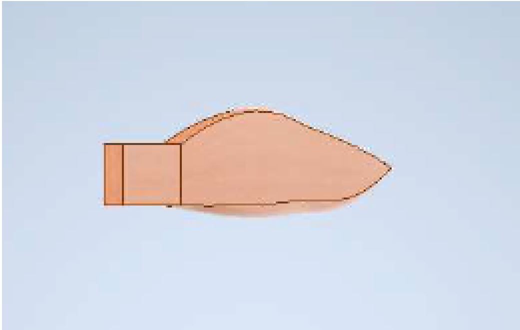
Picture 16. Inventor design (Inventor Software)



Picture 17. Propeller design 1 (Inventor Software)



Picture 18. Propeller design 2 (Inventor Software)



Picture 19. Propeller design 3 (Inventor Software)

4. Wind turbine calculations

4.1 Wind turbine power and energy calculation

Due to our starting data, we have calculated the power that our turbine will produce depending on the efficiency of the turbine.

STARTING DATA	
ROTOR DIAMETER (m)	1
WIND SPEED (m/s)	12
AREA (m2)	0,785375
WIND DENSITY(kg/m³)	1,225

Table 4. Starting data (Excell)

To make this calculation, we need to attend to the following expressions;

$$V = \pi \cdot r^{2} \cdot v$$

$$Ec = \frac{1}{2} \cdot m \cdot v^{2}$$

Where v is the velocity, r the radius, m the mass and Ec the kinetic energy.

4.2. Wind energy per unit mass of air

The first energy that we are going to calcule is the wind energy per unit mass of air:

$$e = ec = \frac{V^{2}}{2} = \frac{12^{2}}{2} \cdot (\frac{1J/kg}{1\,m^{2}/s^{2}}) = 72\,J/kg$$

4.3. Maximum power generation by a wind turbine

Air is at standard conditions of 1atm and 25ºC, and thus its density is 1,225kg/m^{3}. The air flowing with the wind has the same properties as the stagnant atmospheric air except that it possesses a velocity and thus some kinetic energy.

This air will reach the dead state when it is brought to a complete stop. Therefore, the exergy of the blowing air is simply the kinetic energy it possesses:

$$Ke = \frac{V^2}{2} = \frac{(12m/s)^2}{2}\left(\frac{1kJ/kg}{1000m^2/s^2}\right) = 0,072kj/kg$$

That is , every unit mass of air flowing at a velocity of 12 m/s has a work potential of 0,072 kJ/kg . In other words, a perfect wind turbine will bring the air to a complete stop and capture that $0,072kj/kg$ of work potential. To determine the maximum power, we need to know the amount of air passing through the rotor of the wind turbine per unit of time, that is, the mass flow rate, which is determined to be

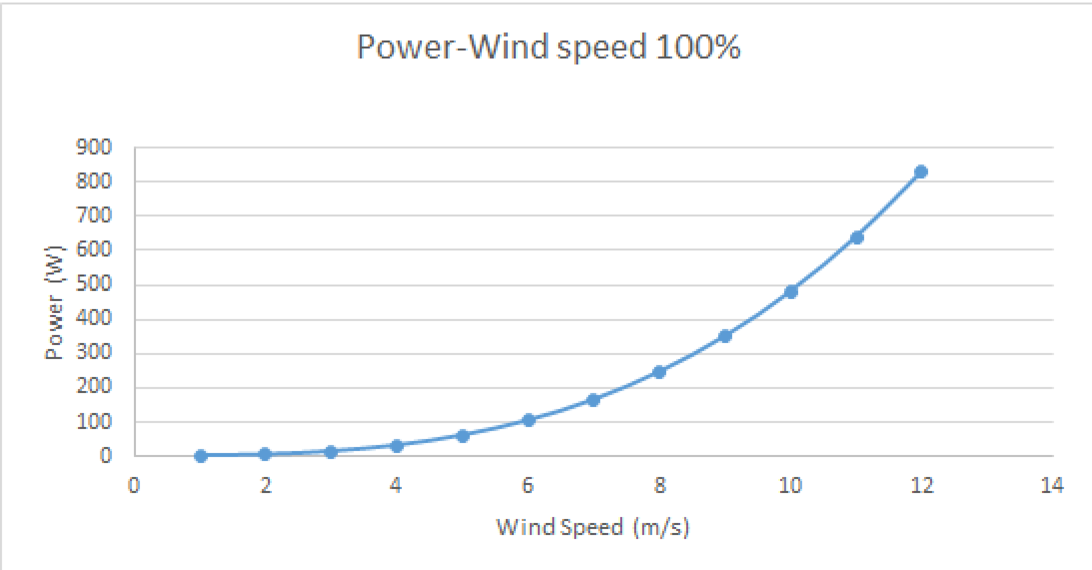
$$\dot{m} = \rho AV = \rho\frac{\pi D^2}{4}V = (1,225\,Kg/m^3)\frac{\pi*(1m)^2}{4} * 12m/s = 11,54Kg/s$$

$$Maximum\,power = \dot{m}(ke) = 11,54kg/s * 0,072kj/kg = 0,83kW$$

This is the maximum available power of the wind turbine. On the following table, I calculated the maximum power of the wind turbine depending on the efficiency and the velocity:

POWER CALCULATION DEPENDING ON THE VELOCITY					
EFFICENCY	WIND SPEED(m/s)	Ke (Kj/Kg)	Mass flow rate (Kg/s)	Maximum power W	Power depending efficency
100%	0	0	0	0	0
	1	0,0005	0,962084375	0,481042188	0,481042188
	2	0,002	1,92416875	3,8483375	3,8483375
	3	0,0045	2,886253125	12,98813906	12,98813906
	4	0,008	3,8483375	30,7867	30,7867
	5	0,0125	4,810421875	60,13027344	60,13027344
	6	0,018	5,77250625	103,9051125	103,9051125
	7	0,0245	6,734590625	164,9974703	164,9974703
	8	0,032	7,696675	246,2936	246,2936
	9	0,0405	8,658759375	350,6797547	350,6797547
	10	0,05	9,62084375	481,0421875	481,0421875
	11	0,0605	10,58292813	640,2671516	640,2671516
	12	0,072	11,5450125	831,2409	831,2409

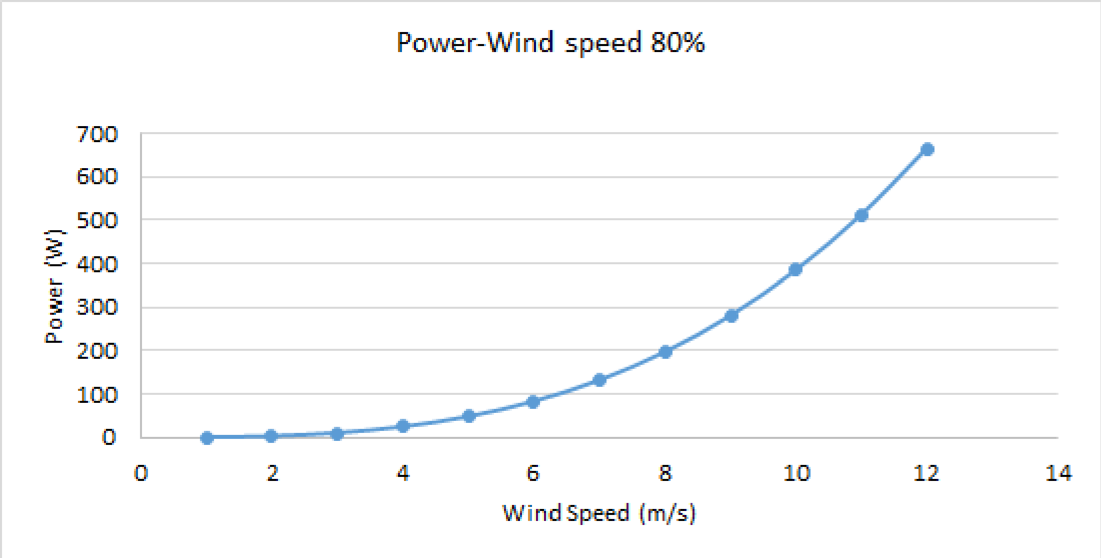
Table 5. Power calculation 100% efficiency



Graph 2. Power calculation 100% efficiency

POWER CALCULATION DEPENDING ON THE VELOCITY					
EFFICENCY	WIND SPEED(m/s)	Ke (Kj/Kg)	Mass flow rate (Kg/s)	Maximum power W	Power depending efficency
80%	0	0	0	0	0
	1	0,0005	0,962084375	0,481042188	0,38483375
	2	0,002	1,92416875	3,8483375	3,07867
	3	0,0045	2,886253125	12,98813906	10,39051125
	4	0,008	3,8483375	30,7867	24,62936
	5	0,0125	4,810421875	60,13027344	48,10421875
	6	0,018	5,77250625	103,9051125	83,12409
	7	0,0245	6,734590625	164,9974703	131,9979763
	8	0,032	7,696675	246,2936	197,03488
	9	0,0405	8,658759375	350,6797547	280,5438038
	10	0,05	9,62084375	481,0421875	384,83375
	11	0,0605	10,58292813	640,2671516	512,2137213
	12	0,072	11,5450125	831,2409	664,99272

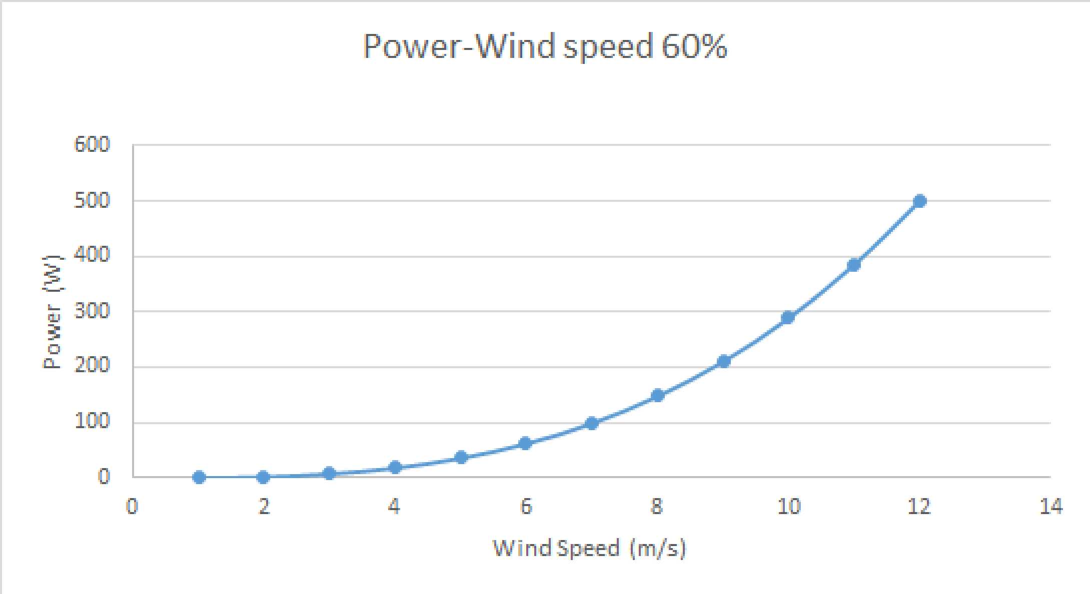
Table 6. Power calculation 80% efficiency



Graph 3. Power calculation 80% efficiency

POWER CALCULATION DEPENDING ON THE VELOCITY					
EFFICENCY	WIND SPEED(m/s)	Ke (Kj/Kg)	Mass flow rate (Kg/s)	Maximum power W	Power depending efficency
60%	0	0	0	0	0
	1	0,0005	0,962084375	0,481042188	0,288625313
	2	0,002	1,92416875	3,8483375	2,3090025
	3	0,0045	2,886253125	12,98813906	7,792883438
	4	0,008	3,8483375	30,7867	18,47202
	5	0,0125	4,810421875	60,13027344	36,07816406
	6	0,018	5,77250625	103,9051125	62,3430675
	7	0,0245	6,734590625	164,9974703	98,99848219
	8	0,032	7,696675	246,2936	147,77616
	9	0,0405	8,658759375	350,6797547	210,4078528
	10	0,05	9,62084375	481,0421875	288,6253125
	11	0,0605	10,58292813	640,2671516	384,1602909
	12	0,072	11,5450125	831,2409	498,74454

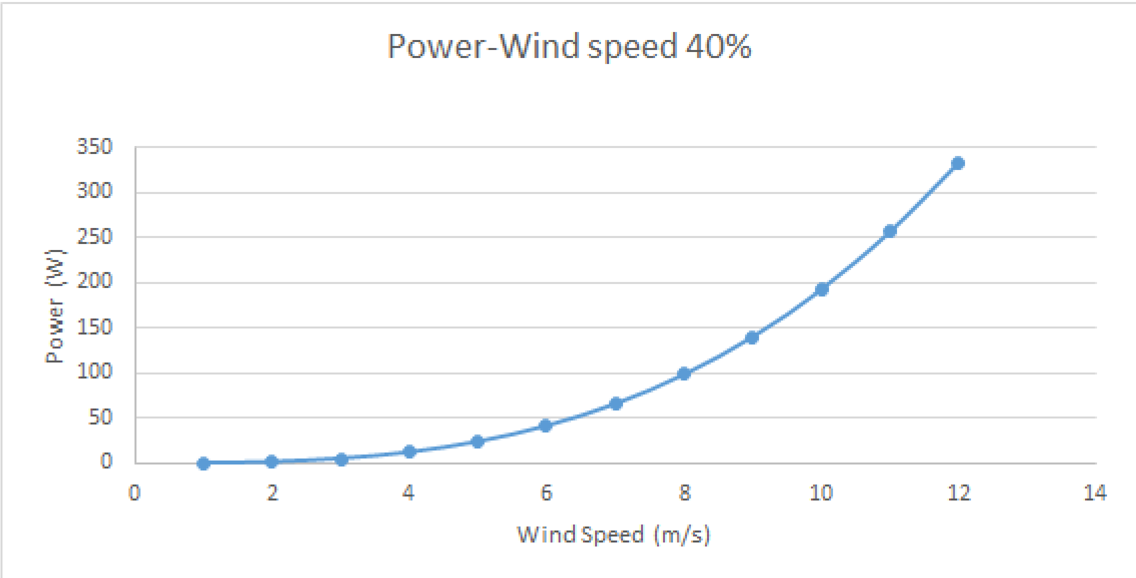
Table 7. Power calculation 60% efficiency



Graph 4. Power calculation 60% efficiency

POWER CALCULATION DEPENDING ON THE VELOCITY					
EFFICENCY	WIND SPEED(m/s)	Ke (Kj/Kg)	Mass flow rate (Kg/s)	Maximum power W	Power depending efficency
40%	0	0	0	0	0
	1	0,0005	0,962084375	0,481042188	0,192416875
	2	0,002	1,92416875	3,8483375	1,539335
	3	0,0045	2,886253125	12,98813906	5,195255625
	4	0,008	3,8483375	30,7867	12,31468
	5	0,0125	4,810421875	60,13027344	24,05210938
	6	0,018	5,77250625	103,9051125	41,562045
	7	0,0245	6,734590625	164,9974703	65,99898813
	8	0,032	7,696675	246,2936	98,51744
	9	0,0405	8,658759375	350,6797547	140,2719019
	10	0,05	9,62084375	481,0421875	192,416875
	11	0,0605	10,58292813	640,2671516	256,1068606
	12	0,072	11,5450125	831,2409	332,49636

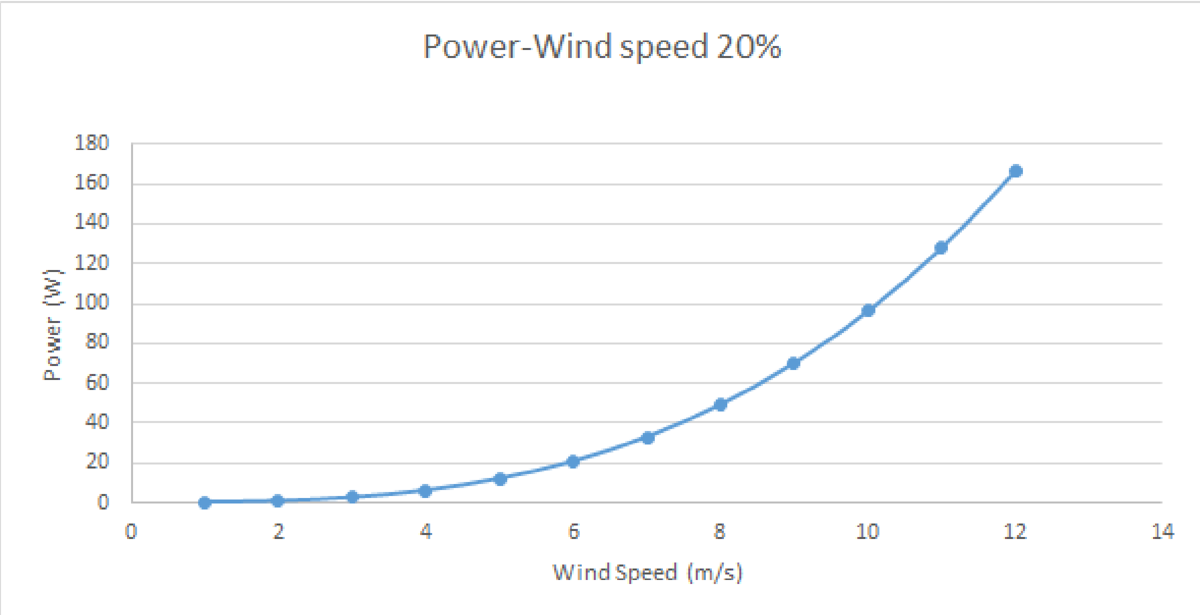
Table 8. Power calculation 40% efficiency



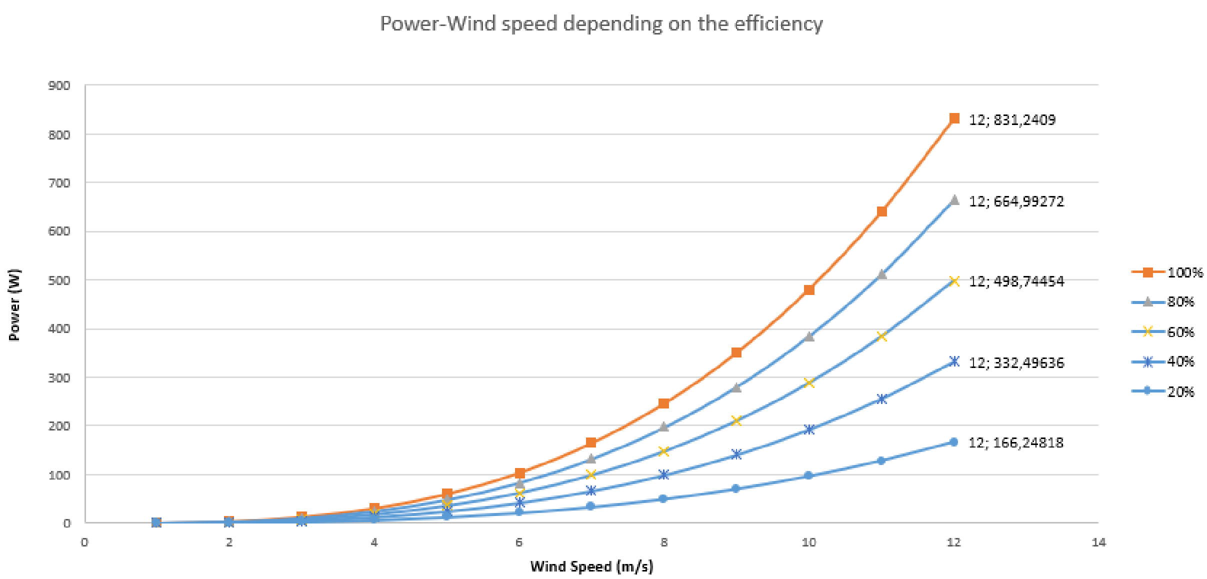
Graph 5. Power calculation 40% efficiency

POWER CALCULATION DEPENDING ON THE VELOCITY					
EFFICENCY	WIND SPEED(m/s)	Ke (Kj/Kg)	Mass flow rate (Kg/s)	Maximum power W	Power depending efficency
20%	0	0	0	0	0
	1	0,0005	0,962084375	0,481042188	0,096208438
	2	0,002	1,92416875	3,8483375	0,7696675
	3	0,0045	2,886253125	12,98813906	2,597627813
	4	0,008	3,8483375	30,7867	6,15734
	5	0,0125	4,810421875	60,13027344	12,02605469
	6	0,018	5,77250625	103,9051125	20,7810225
	7	0,0245	6,734590625	164,9974703	32,99949406
	8	0,032	7,696675	246,2936	49,25872
	9	0,0405	8,658759375	350,6797547	70,13595094
	10	0,05	9,62084375	481,0421875	96,2084375
	11	0,0605	10,58292813	640,2671516	128,0534303
	12	0,072	11,5450125	831,2409	166,24818

Table 9. Power calculation 20% efficiency



Graph 6. Power calculation 20% efficiency



Graph 7. Power calculation general

Betz's law said that the maximum power output of a wind turbine is when the wind is reduced to one third of its initial speed. So the highest efficiency of a wind turbine is approximately 59 percent. In practice, actual efficiency ranges between 20 and 40 percent.

So for our wind turbine we will suppose an **efficiency of 40%** and a wind velocity of **12m/s**. Therefore, our maximum power generated by our prototype is **332,49 W**

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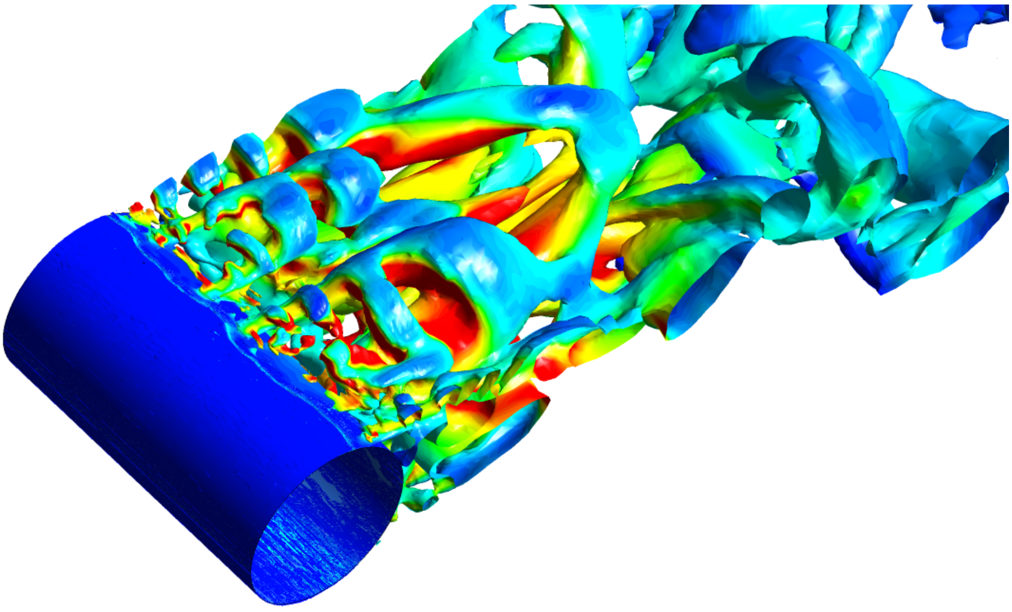
5. Computational fluid dynamics modeling

Turbulence is a fluid movement characterized by chaotic changes in flow velocity and pressure. This occurs in fluid dynamics, as opposed to laminar flow, which occurs when a fluid flows between layers without interruption between them.

Turbulence generally occurs in low-viscosity fluids, because turbulence (caused by excess kinetic energy in parts of a fluid flow) overcomes the damping effect of the viscosity of that fluid.

On the other hand, it is common for unsteady vortices of different sizes to appear in turbulent flows. This increases the resistance due to the effects of friction, thus increasing the energy required for the fluid to be pumped through a pipeline.

Some applications of the above can be in the aerodynamics of vehicles, apart from some other industrial applications, such as heat exchangers, cooling processes, continuous steel casting,...



Reynolds number

The **Reynolds number** is the ratio of inertial forces to viscous forces within a fluid which is subjected to relative internal movement due to different fluid velocities. A region where these forces change behavior is known as a boundary layer, such as the bounding surface in the interior of a pipe.

$$Re = \frac{\rho \nu l}{\mu} = \frac{\nu l}{\nu}$$

Where V is the velocity of the fluid, l is the characteristic length, the chord width of an airfoil, ρ the density of the fluid and μ the dynamic viscosity of the fluid.

5.1. Turbulence models in CFD

Turbulence modeling is a key issue in most CFD simulations. Virtually all engineering applications are turbulent and hence require a turbulence model.

Two equation turbulence models are one of the most common types of turbulence models. Models like the **k-epsilon** model and the **k-omega** model have become industry standard models and are commonly used for most types of engineering problems. Two equation turbulence models are also very much still an active area of research and new refined two-equation models are still being developed.

By definition, two equation models include two extra transport equations to represent the turbulent properties of the flow. This allows a two equation model to account for history effects like convection and diffusion of turbulent energy.

5.1.1. K-epsilon models

The K-epsilon model is one of the most common turbulence models, although it just doesn't perform well in cases of large adverse pressure gradients. It is a two equation model, that means, it includes two extra transport equations to represent the turbulent properties of the flow. This allows a two equation model to account for history effects like convection and diffusion of turbulent energy.

Standard k-epsilon model

For turbulent **kinetic energy**

$$\frac{\delta}{\delta t}(\rho k) + \frac{\delta}{\delta Xi}(\rho k u i) = \frac{\delta}{\delta Xj}\left[\left(\mu + \frac{\mu t}{\sigma k}\right)\frac{\delta k}{\delta Xj}\right] + P_k + P_b - \rho\epsilon - Y_M + S_K$$

For dissipation ϵ

$$\frac{\delta}{\delta t}(\rho\epsilon) + \frac{\delta}{\delta Xi}(\rho\epsilon ui) = \frac{\delta}{\delta Xj}\left[\left(\mu + \frac{\mu t}{\sigma\epsilon}\right)\frac{\delta\epsilon}{\delta Xj}\right] + C1\epsilon\frac{\epsilon}{k}(P_k + C3\epsilon P_b) - C2\epsilon\rho\frac{\epsilon^2}{k} + S\epsilon$$

Turbulent viscosity is modelled as:

$$\mu t = \rho C\mu\frac{k^2}{\epsilon}$$

Production of K

$$P_k = -\rho\overline{u_i'u_j'}\frac{\partial u_j}{\partial x_i}$$
$$P_k = \mu_t S^2$$
$$S \equiv \sqrt{2S_{ij}S_{ij}}$$

Effect of buoyancy

$$P_b = \beta g_i \frac{\mu_t}{\mathrm{Pr}_t}\frac{\partial T}{\partial x_i}$$

$$\beta = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p$$

It is important to know the **model constants**
$C_{1\epsilon} = 1.44, C_{2\epsilon} = 1.92, C_{\mu} = 0.09, \sigma k = 1.0, \sigma\epsilon = 1.3$

Realisable k-epsilon model

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho k u_j) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P_k + P_b - \rho\epsilon - Y_M + S_k$$

$$\frac{\partial}{\partial t}(\rho\epsilon)+\frac{\partial}{\partial x_j}(\rho\epsilon u_j) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\epsilon}\right)\frac{\partial\epsilon}{\partial x_j}\right]+\rho C_1 S\epsilon-\rho C_2\frac{\epsilon^2}{k+\sqrt{\nu\epsilon}}+C_{1\epsilon}\frac{\epsilon}{k}C_{3\epsilon}P_b+S_\epsilon$$

Where

$$C_1 = \max\left[0.43, \frac{\eta}{\eta+5}\right], \quad \eta = S\frac{k}{\epsilon}, \quad S = \sqrt{2S_{ij}S_{ij}}$$

$$\mu_t = \rho C_\mu \frac{k^2}{\epsilon}$$

where

$$C_\mu = \frac{1}{A_0 + A_s \frac{kU^*}{\epsilon}}$$

$$U^* \equiv \sqrt{S_{ij}S_{ij} + \tilde{\Omega}_{ij}\tilde{\Omega}_{ij}};$$

$$\tilde{\Omega}_{ij} = \Omega_{ij} - 2\epsilon_{ijk}\omega_k;$$

$$\Omega_{ij} = \overline{\Omega_{ij}} - \epsilon_{ijk}\omega_k$$

where $\overline{\Omega_{ij}}$ is the mean rate-of-rotation tensor viewed in a rotating reference frame with the angular velocity ω_k. The model constants A_0 and A_s are given by:

$$A_0 = 4.04, \quad A_s = \sqrt{6}\cos\phi$$

$$\phi = \frac{1}{3}\cos^{-1}(\sqrt{6}W), \quad W = \frac{S_{ij}S_{jk}S_{ki}}{\tilde{S}^3}, \quad \tilde{S} = \sqrt{S_{ij}S_{ij}}, \quad S_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right)$$

It is important to know the **model constants**

$C_{1\epsilon} = 1.44, C_2 = 1.9, \sigma k = 1.0, \sigma\epsilon = 1.2$

RNG k-epsilon model

There are a number of ways to write the transport equations for k and ϵ, a simple interpretation where bouyancy is neglected is

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P_k - \rho\epsilon$$

$$\frac{\partial}{\partial t}(\rho \epsilon) + \frac{\partial}{\partial x_i}(\rho \epsilon u_i) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\epsilon}\right)\frac{\partial \epsilon}{\partial x_j}\right] + C_{1\epsilon}\frac{\epsilon}{k}P_k - C^*_{2\epsilon}\rho\frac{\epsilon^2}{k}$$

where $C^*_{2\epsilon} = C_{2\epsilon} + \frac{C_\mu \eta^3(1 - \eta/\eta_0)}{1 + \beta\eta^3}$

and $\eta = Sk/\epsilon$ and $S = (2S_{ij}S_{ij})^{1/2}$

With the turbulent viscosity being calculated in the same manner as with the standard k-epsilon model.

It is important to know the **model constants**

$C\mu = 0.0845\ (0.09)$

$\sigma k = 0.7194(1.0)$

$\sigma\epsilon = 0.7194\ (1.30)$

$C_{\epsilon 1} = 1.42(1.44)$

$C_{\epsilon 2} = 1.68(1.92)$

$\eta 0 = 4.38$

$\beta = 0.012(derived\ from\ experiment)$

5.1.2. K-omega models

The K-omega model is one of the most commonly used turbulence models. It is a two equation model, that means, it includes two extra transport equations to represent the turbulent properties of the flow. This allows a two equation model to account for history effects like convection and diffusion of turbulent energy.

The first transported variable is turbulent kinetic energy, k. The second transported variable in this case is the specific dissipation, omega. It is the variable that determines the scale of the turbulence, whereas the first variable, k, determines the energy in the turbulence.

Wilcox's k-omega model

Kinematic Eddy Viscosity

$$vt = \frac{k}{w}$$

Turbulence kinetic energy

$$\frac{\delta k}{\delta t} + Uj\frac{\delta k}{\delta xj} = \tau ij\frac{\delta Ui}{\delta xj} - \beta * kw + \frac{\delta}{\delta xj}\left[(v + \sigma * vt)\frac{\delta k}{\delta xj}\right]$$

Specific dissipation rate

$$\frac{\delta w}{\delta t} + Uj\frac{\delta w}{\delta xj} = \alpha\frac{w}{k}\tau ij\frac{\delta Ui}{\delta xj} - \beta w^{2} + \frac{\delta}{\delta xj}\left[(v + \sigma * vt)\frac{\delta w}{\delta xj}\right]$$

Closure Coefficients and Auxiliary Relations

$$\alpha = \frac{5}{9}$$

$$\beta = \frac{3}{40}$$

$$\beta * = \frac{9}{100}$$

$$\sigma = \frac{1}{2}$$

$$\sigma * = \frac{1}{2}$$

$$\varepsilon = \beta * \omega k$$

5.1.3. SST k-omega model

The SST k-ω turbulence model [Menter 1993] is a two-equation eddy-viscosity model which has become very popular. The shear stress transport (SST) formulation combines the best of two worlds. The use of a k-ω formulation in the inner parts of the boundary layer makes the model directly usable all the way down to the wall through the viscous sub-layer, hence the SST k-ω model can be used as a Low-Re turbulence model without any extra damping functions. The SST formulation also switches to a k-ε behaviour in the free-stream and thereby avoids the common k-ω problem that the model is too sensitive to the inlet free-stream turbulence properties. Authors who use the SST k-ω model often merit it for its good behaviour in adverse pressure gradients and separating flow. The SST k-ω model does produce a bit too large turbulence levels in regions with large normal strain, like stagnation regions and regions with strong acceleration. This tendency is much less pronounced than with a normal k-ε model though.

Kinematic Eddy Viscosity

$$vt = \frac{a1K}{max(a1\omega, SF2)}$$

Turbulence kinetic energy

$$\frac{\delta k}{\delta t} + Uj\frac{\delta k}{\delta xj} = Pk - \beta * k\omega + \frac{\delta}{\delta xj}\left[(v + \sigma kvt)\frac{\delta k}{\delta xj}\right]$$

Specific dissipation rate

$$\frac{\delta w}{\delta t} + Uj\frac{\delta w}{\delta xj} = \alpha S^{2} - \beta\omega^{2} + \frac{\delta}{\delta xj}\left[(v + \sigma\omega vt)\frac{\delta w}{\delta xj}\right] + 2(1 - F1)\sigma_{\omega 2}\frac{1}{\omega}\frac{\delta k}{\delta xi}\frac{\delta w}{\delta xi}$$

Closure Coefficients and Auxiliary Relations

$$F2 = tanh\left[\left[max\left(\frac{2\sqrt{K}}{\beta^{*}\omega y}, \frac{500v}{y^{2}\omega}\right)\right]^{2}\right]$$

$$Pk = min\left(\tau ij\frac{\delta Ui}{\delta xj}, 10\beta * k\omega\right)$$

$$F1 = tanh\left\{\left\{min\left[max\left(\frac{\sqrt{K}}{\beta^{*}\omega y}, \frac{500v}{y^{2}\omega}\right), \frac{4\sigma w2k}{CD_{kw}y^{2}}\right]\right\}^{4}\right\}$$

$$CD_{kw} = max\left(2\rho\sigma\omega 2\frac{1}{w}\frac{\delta k}{\delta xi}\frac{\delta w}{\delta xi}, 10^{-10}\right)$$

$$\phi = \phi 1F1 + \phi 2(1 - F1)$$

$$\alpha 1 = \frac{5}{9}, \alpha 2 = 0.44$$

$$\beta 1 = \frac{3}{40}, \beta 2 = 0.0828$$

$$\beta * = \frac{9}{100}$$

$$\sigma k1 = 0.85, \sigma k2 = 1$$

$$\sigma\omega 1 = 0.5, \sigma\omega 2 = 0.856$$

5.1.4. Large eddy simulation (LES)

Large eddy simulation (LES) is a popular technique for simulating turbulent flows. An implication of Kolmogorov's (1941) theory of self similarity is that the large eddies of the flow are dependent on the geometry while the smaller scales are more universal. This feature allows one to explicitly solve for the large eddies in a calculation and implicitly account for the small eddies by using a subgrid-scale model (SGS model).

Mathematically, one may think of separating the velocity field into a resolved and sub-grid part. The resolved part of the field represents the "large" eddies, while the subgrid part of the velocity represents the "small scales" whose effect on the resolved field is included through the subgrid-scale model. Formally, one may think of filtering as the convolution of a function with a filtering kernel G:

$$\bar{u}_i(\vec{x}) = \int G(\vec{x} - \vec{\xi}) u(\vec{\xi}) d\vec{\xi},$$

resulting in

$$u_i = \bar{u}_i + u_i',$$

The filtered equations are developed from the incompressible **Native-Stokes equations** of motion

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\nu \frac{\partial u_i}{\partial x_j}\right).$$

Substituting in the decomposition $u_i = \bar{u}_i + u_i'$ and $p = \bar{p} + p'$ and then filtering the resulting equation gives the equations of motion for the resolved field:

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\nu \frac{\partial \bar{u}_i}{\partial x_j}\right) + \frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_j}.$$

$$\overline{u_j \frac{\partial u_i}{\partial x_j}} \neq \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} \qquad \tau_{ij} = \bar{u}_i \bar{u}_j - \overline{u_i u_j}$$

$$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -2\mu_t \bar{S}_{ij}$$

$$\bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$$

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left([\nu + \nu_t]\frac{\partial \bar{u}_i}{\partial x_j}\right),$$

Where we have used the **incompressibility constraint** to simplify the equation and the pressure is now modified to include the trace term $\tau_{kk}\delta_{ij}/3$.

5.2. Parameter Y+

Y+ is an important parameter to be considered in any CFD simulation for good results. Here is formulae for Y+ and related terms:

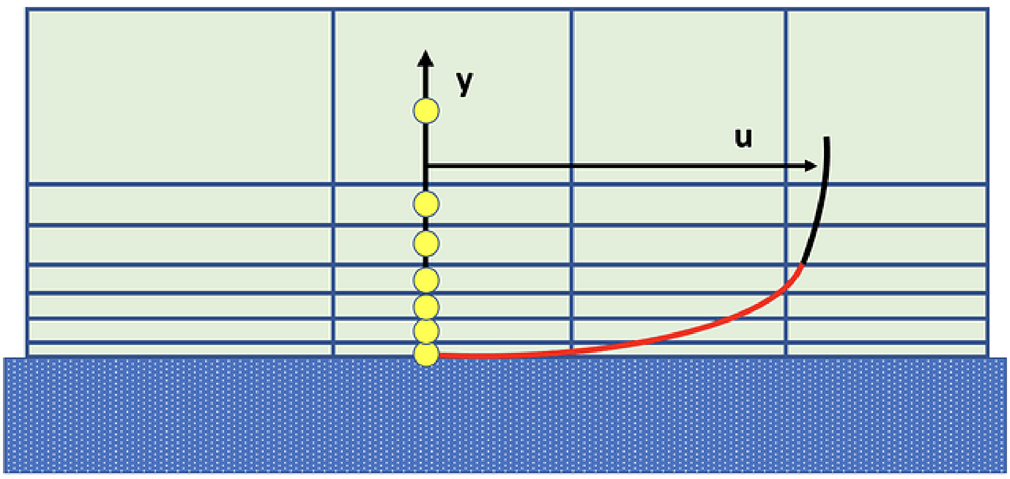
$$Y += \frac{\mu t \cdot \Delta y 1 \cdot \rho}{\mu}$$

$\mu t = \sqrt{\frac{tw}{\rho}}$ Friction velocity formula

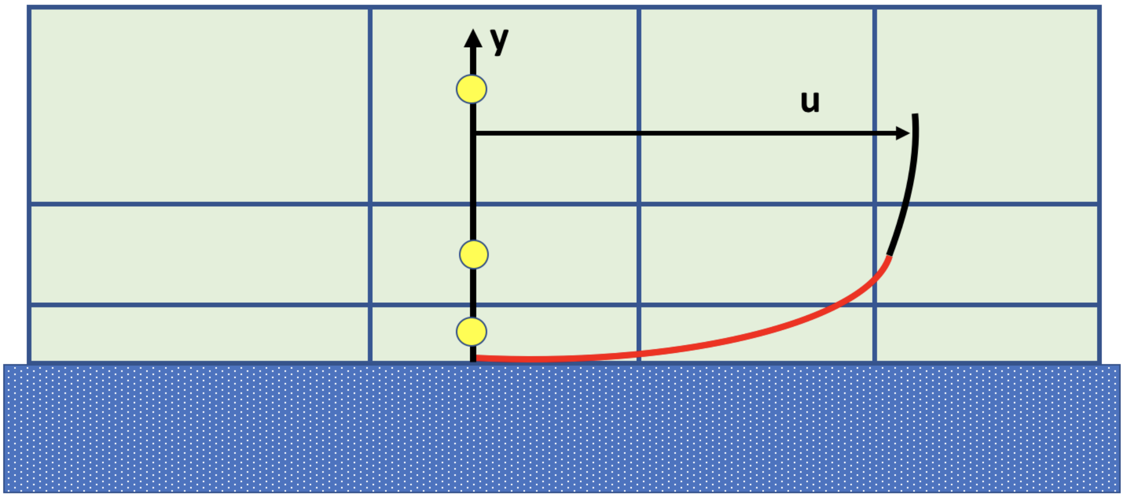
$tw = 0.5 Cf\, \rho\mu^{2}$ Wall shear stress formula

$Cf = \frac{0.058}{Re^{0.2}}$ Correlation to find out skin friction coefficient

Where Tw is the thickness, ρ is the density, Cf the friction coefficient and Re the reynolds number.



Graph 8. Y+ explanation 1 (https://www.simscale.com/)

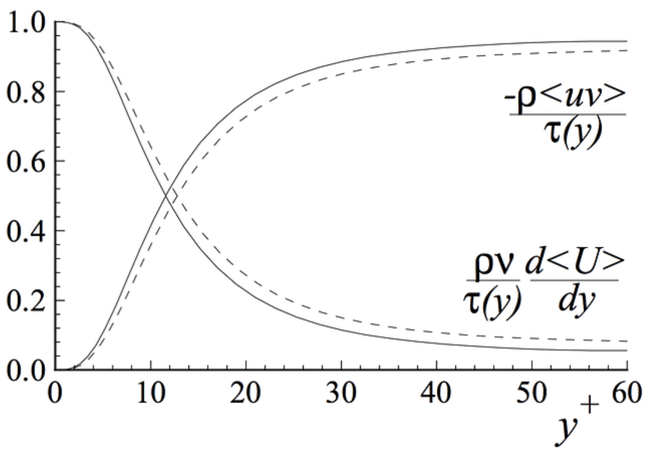


Graph 9. Y+ explanation 2 (https://www.simscale.com/)

The wall functions rely on the universal law of the wall, which basically states that the velocity distribution very near to a wall is similar for almost all turbulent flows. One of the most prominent parameters when judging the applicability of wall functions is the so-called dimensionless wall distance y+

$$y^{+} = \frac{y\mu_{t}}{\nu}$$

Where μt is the friction velocity, y is the absolute distance of the wall and ν is the kinematic viscosity



Graph 10. Y+ Esplanation 3 (https://www.simscale.com/)

Step-by-Step Calculating wall distance with a given Y+

1. What is y+ and rearrange for y

$$y^{+} = \frac{y\mu_{t}}{\nu} \rightarrow y = \frac{y^{+}\nu}{\mu t}$$

2. Calculate μt

$$\mu t = \sqrt{\frac{Tw}{\rho}}$$

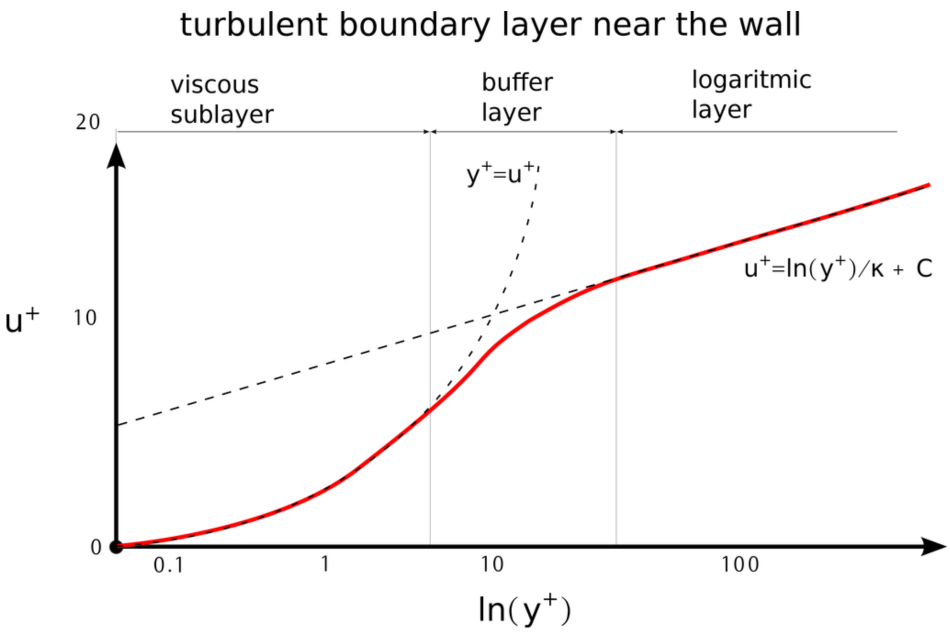
3. Calculate shear stress Tw with the skin friction coefficient Cf

$$Tw = \frac{1}{2} Cf \rho U^{2}{}_{\infty}$$

4. Determine the skin friction coefficient Cf

$$Cf = 0.0576 Re_{d}^{-\frac{1}{5}}$$

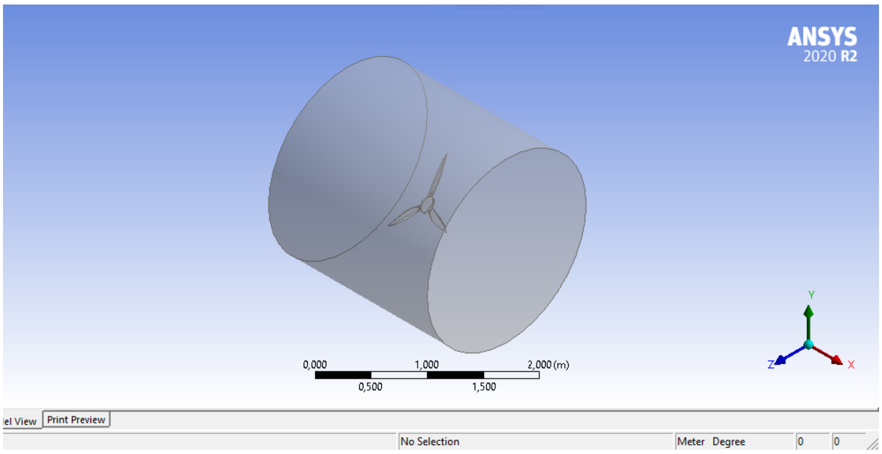
5. Insert reynolds number on the equations show before and complete with numerical numbers



Graph 11. Turbulent boundary layer near the wall (https://www.linkedin.com/pulse/understanding-y-cfd-simulation-sijal-ahmed/)

Y+ is a non-dimensional distance. It is often used to describe how coarse or fine a mesh is for a particular flow pattern. It is important in turbulence modeling to determine the proper size of the cells near domain walls. The turbulence model wall laws have restrictions on the y+ value at the wall. For instance, the standard K-epsilon model requires a wall y+ value between approximately 300 and 100. A faster flow near the wall will produce higher values of y+, so the grid size near the wall must be reduced.

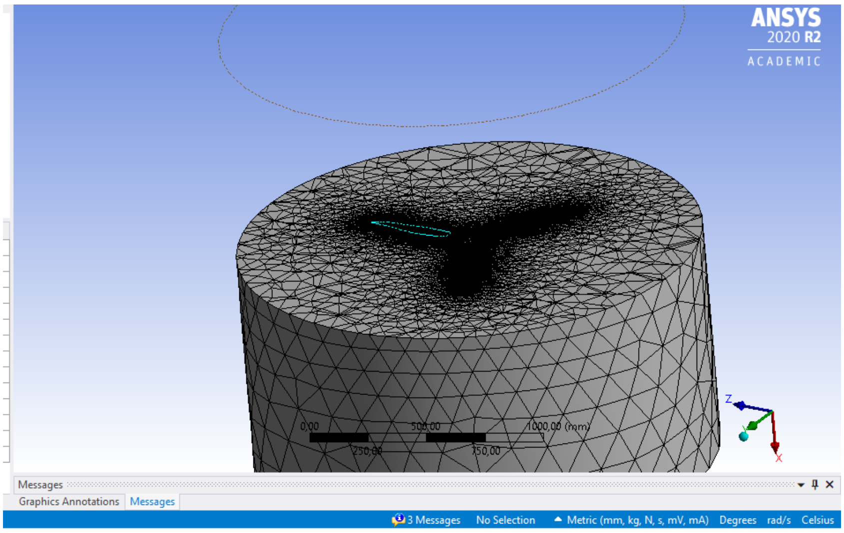
5.3. Geometry construction



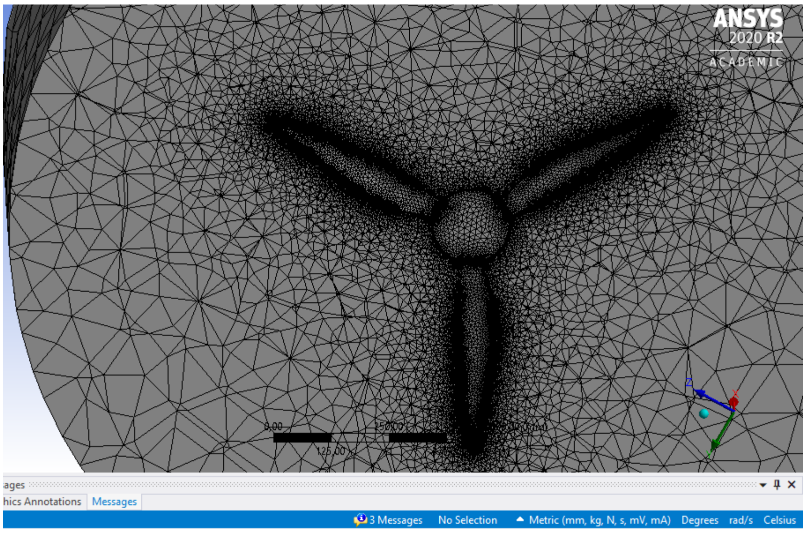
Picture 20. Geometry construction (ANSYS Software)

5.4. Mesh settings

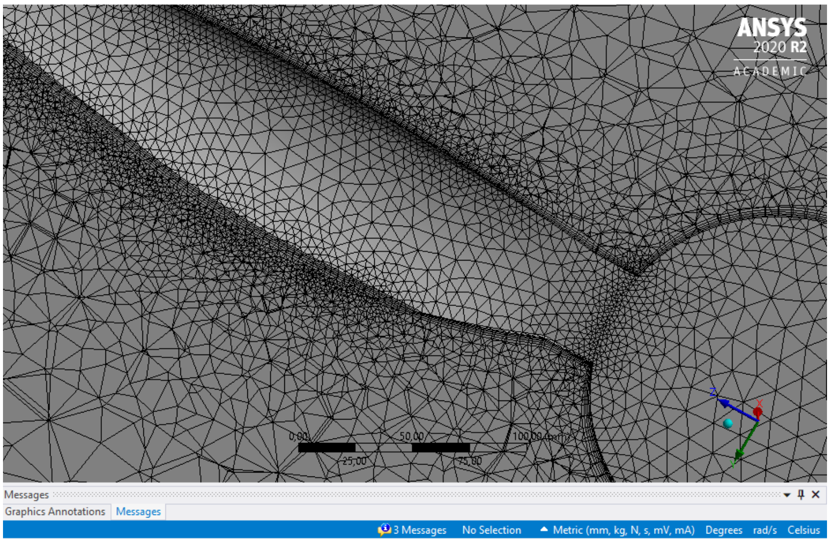
For the mesh settings, after a large number of experiments changing the data to make the software work, I used the following configuration, **inflation mode** on the turbine design and an element side of **50 mm.** The **number of nodes is 190915** and the **number of elements is 848022.** I also activated the **capture proximity mode.**



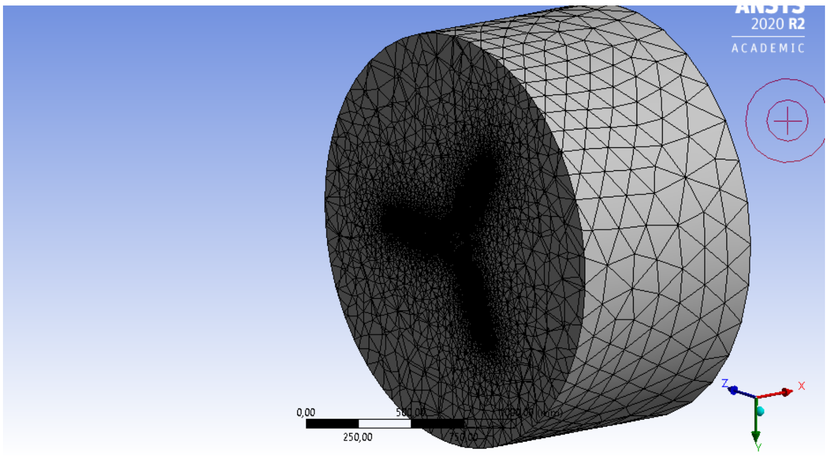
Picture 21. Mesh settings 1 (ANSYS Software)



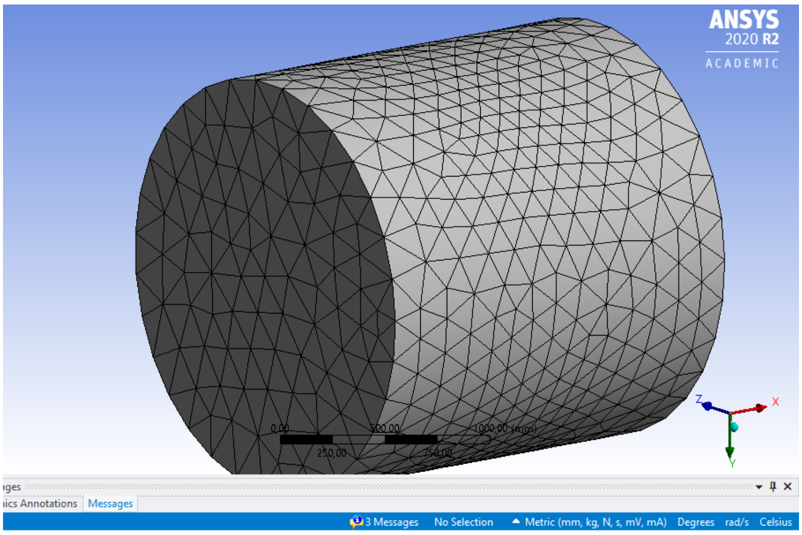
Picture 22. Mesh settings 2 (ANSYS Software)



Picture 23. Mesh settings 3 (ANSYS Software)



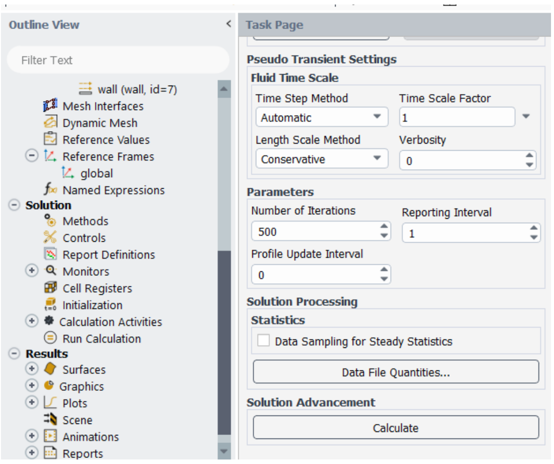
Picture 24. Mesh settings 4 (ANSYS Software)

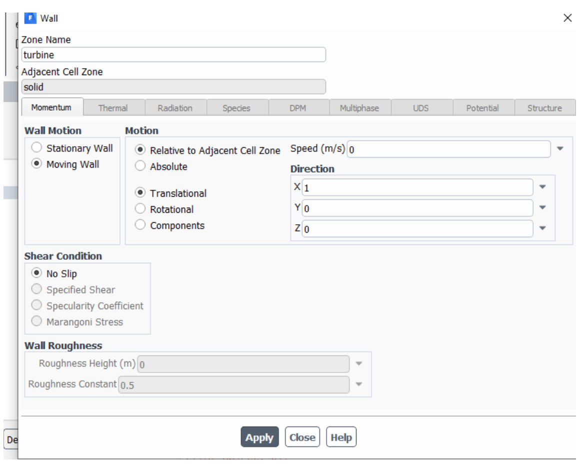


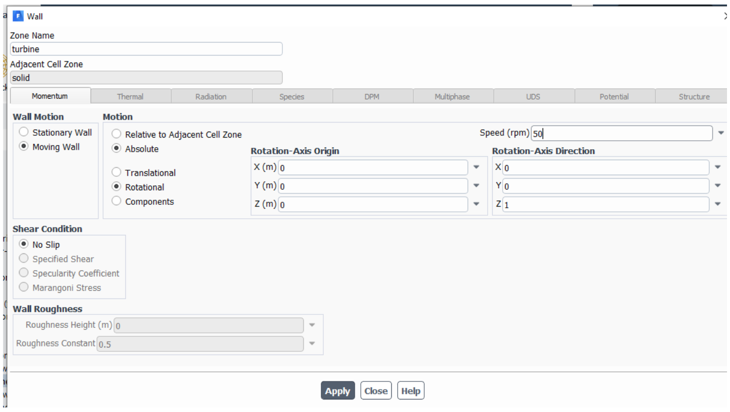
Picture 25. Mesh settings 5 (ANSYS Software)

5.5. Results

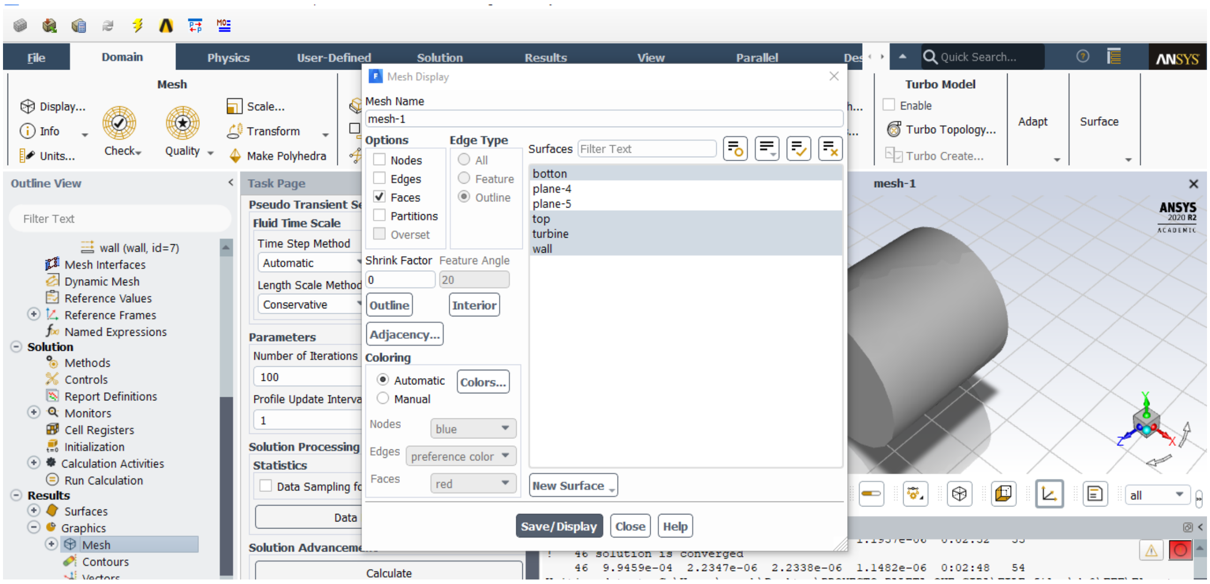
After making all the designs and the meshing and geometry setups, I continue with the results model from ANSYS fluent. To simulate my project there, I started settings some starting data as we can see on the screenshots:

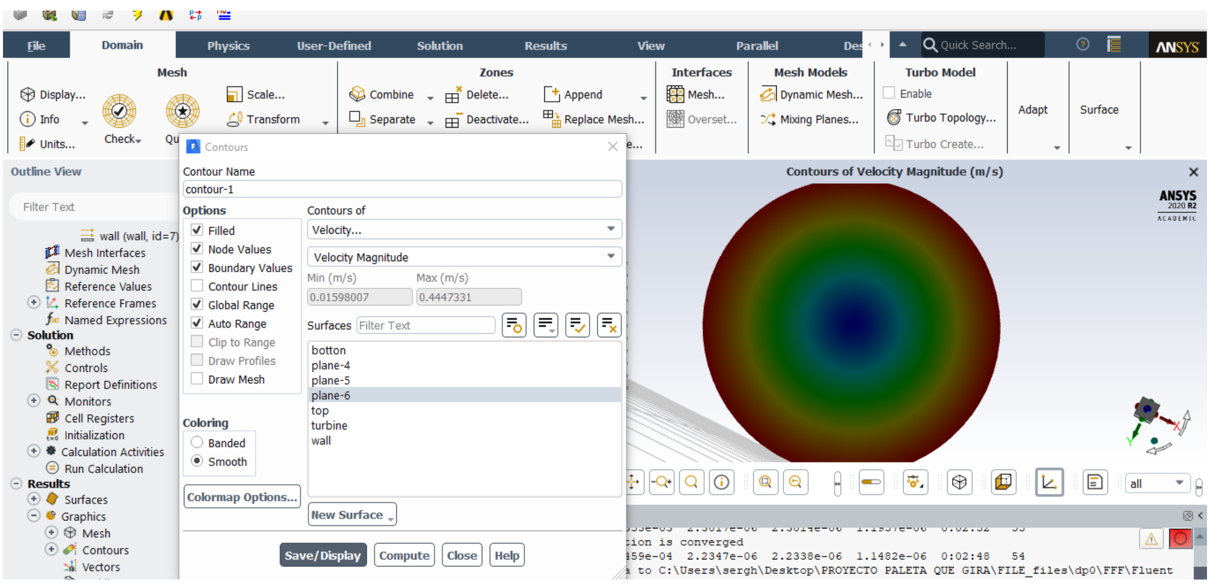






To obtain the most accurate results, I decided to create different planes to compare it. The parts of our turbine that we are going to study are: The **top**, the **button**, the **turbine**, the **wall** and a **plane** that is very close to the turbine and gives us the velocity of the air on that plane.

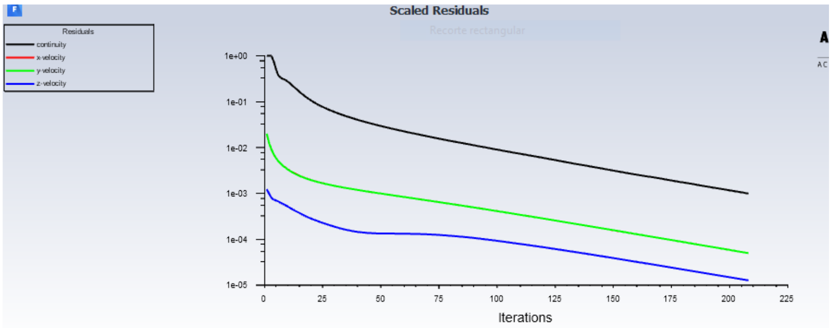




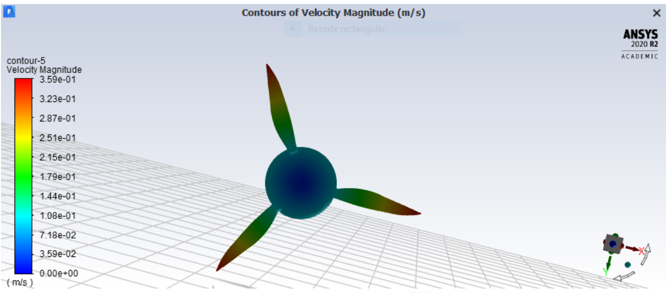
After doing some test simulations, I have been able to verify that the rotational speed that is closest to the speed of our wind tunnel is 500 rpm, because if I exceed that speed the program is not able to recalculate the results. That is why I decided to do 3 simulations to check the speed range of our turbine, **50 RPM, 250 RPM and 500 RPM.**

In the following chapter of this report I attached some screenshots of the program results. Finally, I attached the results of the momentum and the velocity on the plane that I created before.

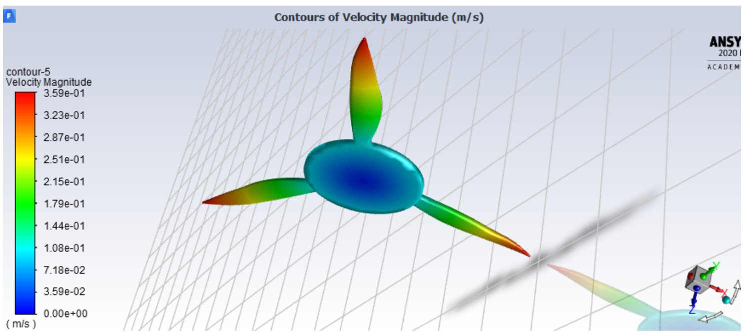
50 RPM



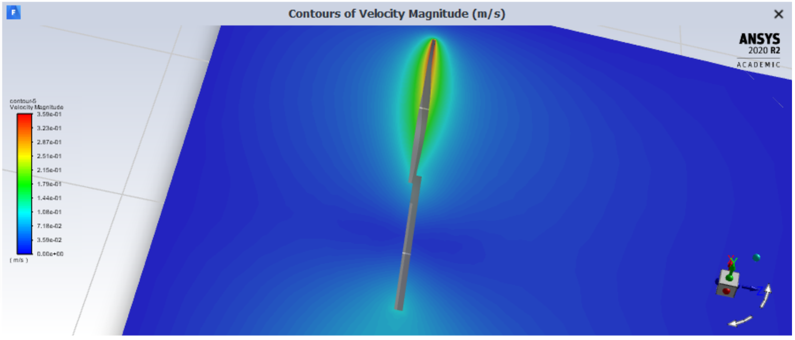
Graph 12. Scaled residuals 50rpm (ANSYS Software)



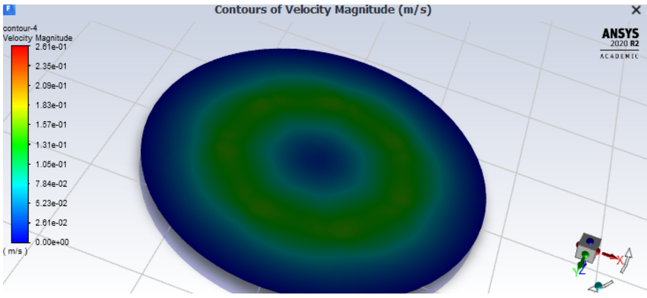
Picture 26. Contour print 50 rpm (ANSYS Software)



Picture 27. Contour print 50 rpm (ANSYS Software)



Picture 28. Contour print 50 rpm (ANSYS Software)



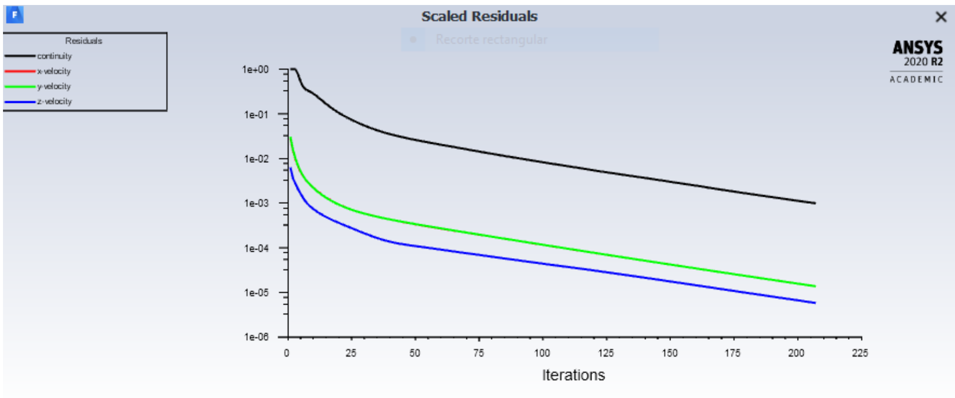
Picture 29. Contour print 50 rpm (ANSYS Software)

Console

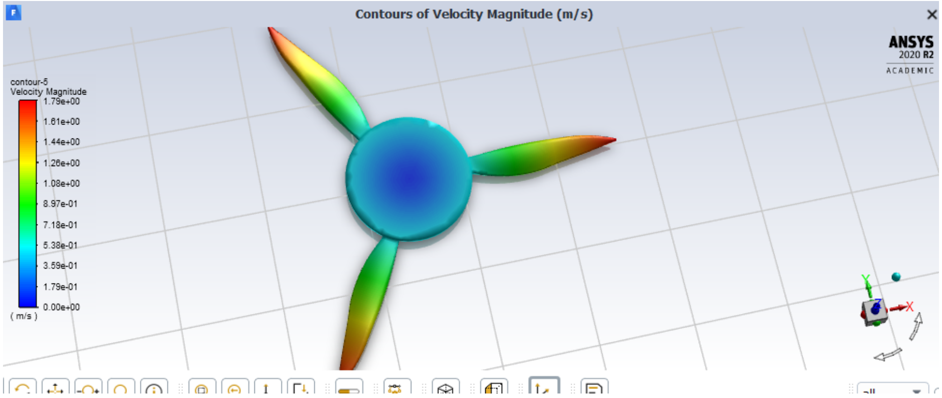
	Moments (n-m)			Coefficients		
Zone	Pressure	Viscous	Total	Pressure	Viscous	Total
turbine	-1.2396486e-06	1.4141624e-06	1.7451384e-07	-2.023916e-06	2.3088366e-06	2.8492056e-07
Net	-1.2396486e-06	1.4141624e-06	1.7451384e-07	-2.023916e-06	2.3088366e-06	2.8492056e-07

Table 10. Results 50 rpm (ANSYS Software)

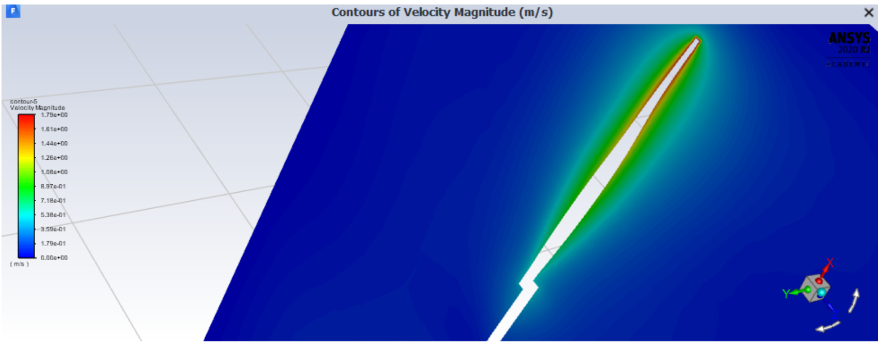
250 RPM



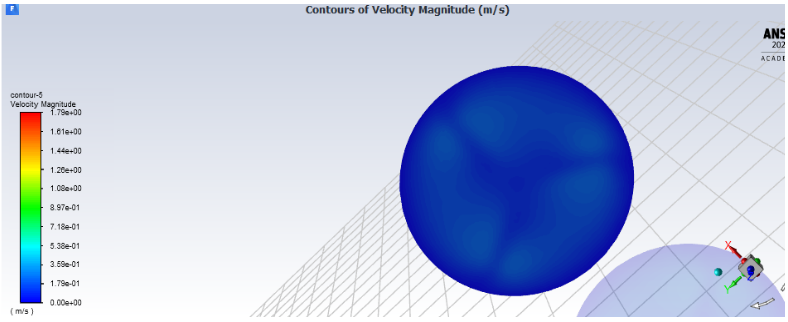
Graph 13. Scaled residuals 250rpm (ANSYS Software)



Picture 30. Contour print 250 rpm (ANSYS Software)



Picture 31. Contour print 250 rpm (ANSYS Software)



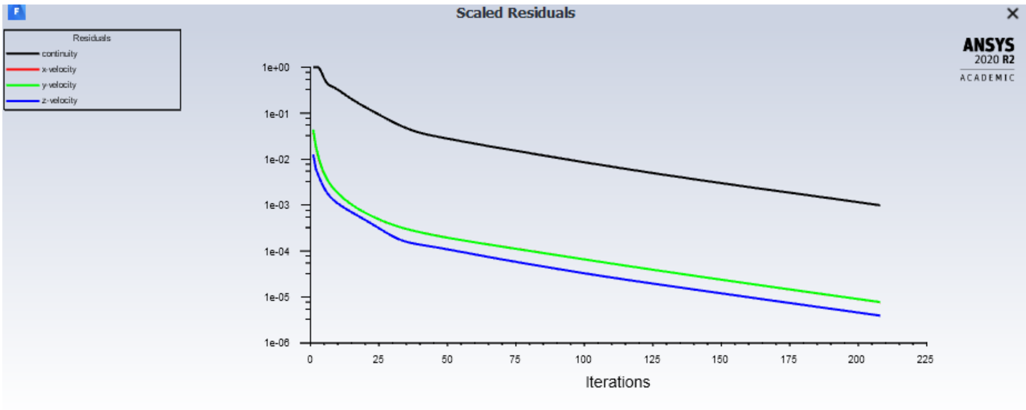
Picture 32. Contour print 250 rpm

Console

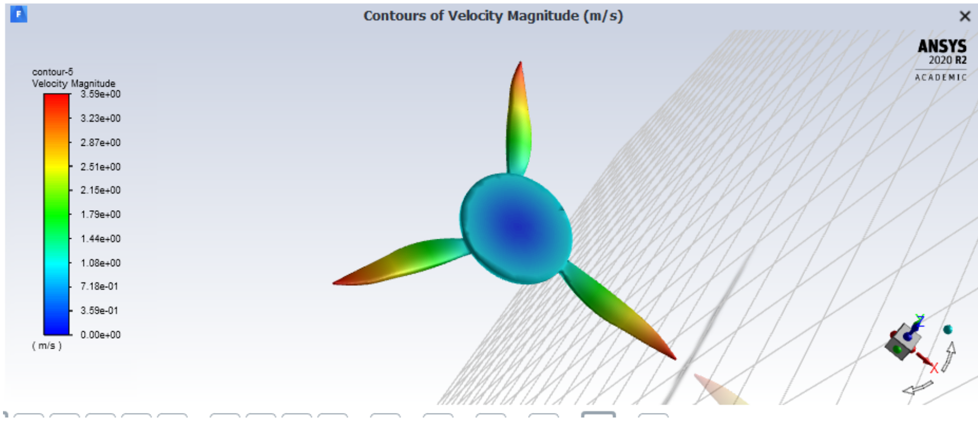
Zone	Moments (n-m) Pressure	Viscous	Total	Coefficients Pressure	Viscous	Total
turbine	1.3676821e-05	-2.1428859e-05	-7.7520381e-06	2.2329504e-05	-3.4985893e-05	-1.2656389e-05
Net	1.3676821e-05	-2.1428859e-05	-7.7520381e-06	2.2329504e-05	-3.4985893e-05	-1.2656389e-05

Table 11. Results 250 rpm (ANSYS Software)

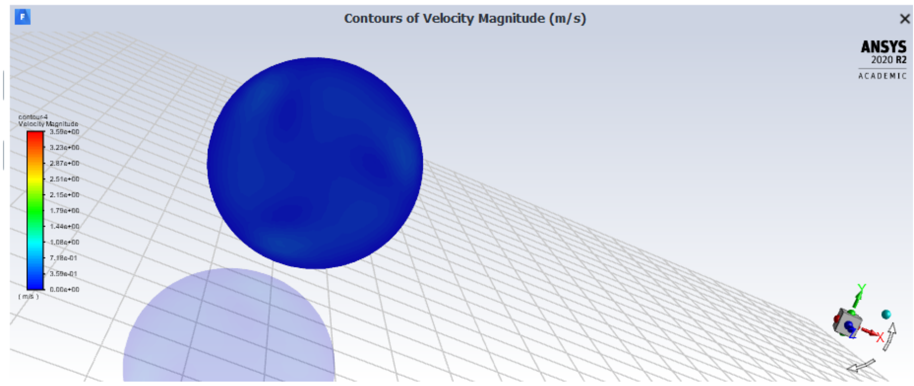
500 RPM



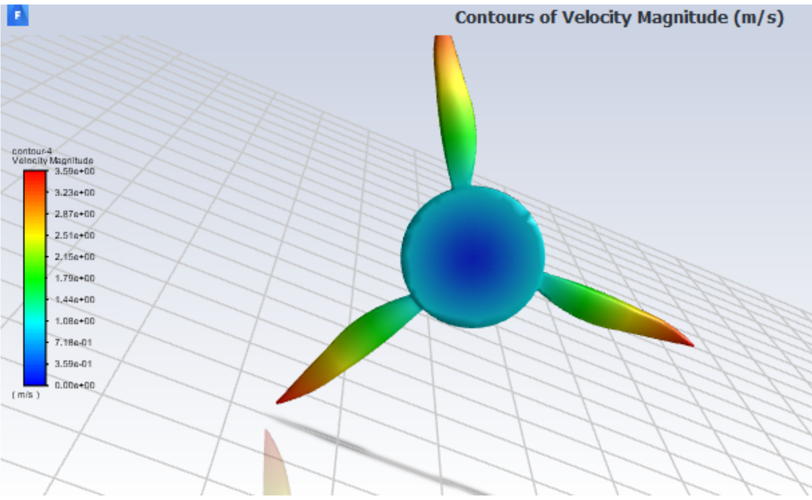
Graph 14. Scaled residuals 500rpm (ANSYS Software)



Picture 33. Contour print 500 rpm (ANSYS Software)



Picture 34. Contour print 500 rpm (ANSYS Software)



Picture 35. Contour print 500 rpm (ANSYS Software)

Console

	Moments (n-m)			Coefficients		
Zone	Pressure	Viscous	Total	Pressure	Viscous	Total
turbine	2.5784364e-05	-5.2197604e-05	-2.641324e-05	4.2096921e-05	-8.5220577e-05	-4.3123656e-05
Net	2.5784364e-05	-5.2197604e-05	-2.641324e-05	4.2096921e-05	-8.5220577e-05	-4.3123656e-05

Table 12. Results 500 rpm (ANSYS Software)

To sum up, I decided to regroup all the results on the following table.

RPM	MOMENTUM(n-m)	ITERATIONS	VELOCITY PLANE(m/s)
50	$-1,23X10^{-6}$	208	1,182
250	$1,36X10^{-5}$	206	4,259
500	$2,57X10^{-5}$	212	8,624

Table 13. Results summary

As we can see, as the RPM is higher, the velocity on the plane changes and the momentum also increases. The numerical sign depends on the side that the turbine is rotating.

6. Discussion and Concluding Remarks

A design and simulation of a wind turbine is developed in the present report. In the first chapters of this document, I have tried to collect all the information about wind turbines. In it, we have dealt with the beginning of wind turbines to the present day of wind turbines. This has helped us to understand how they work and how they have evolved throughout history.

In the following chapters of this document, I have designed with the help of CAD software a wind turbine blade for our design. After having found a design that suits our needs I have exported it to a flow simulation program to simulate its behavior in wind tunnel conditions that we have at the university.

When using the flow simulation software, I have had several problems with it because it was the first time I used it. In the mesh calculation part, I had to adjust the element size to 50mm, since a smaller size did not allow me to obtain the calculations. On the other hand, at the time of finishing the calculations in the fluent section I have decided to make the calculations for 3 speeds, 50 rpm, 250 rpm and 500 rpm, so that the speed of the turbine is adjusted to the speed of the tunnel that we have in the university.

Finally, after obtaining the table of results we can verify that our design works well at speeds of **500 rpm**, reaching a wind speed of about **8.6 m/s**. Our tunnel gave us in ideal conditions a wind speed of 12m/s. Comparing the results we have obtained with those calculated in the energy section, we can say that our design has an efficiency percentage of **40%** and an output of **246.49 W**. *(All this data are obtained from the table 8)*

POWER CALCULATION DEPENDING ON THE VELOCITY					
EFFICENCY	WIND SPEED(m/s)	Ke (Kj/Kg)	Mass flow rate (Kg/s)	Maximum power W	Power depending efficency
40%	0	0	0	0	0
	1	0,0005	0,962084375	0,481042188	0,192416875
	2	0,002	1,92416875	3,8483375	1,539335
	3	0,0045	2,886253125	12,98813906	5,195255625
	4	0,008	3,8483375	30,7867	12,31468
	5	0,0125	4,810421875	60,13027344	24,05210938
	6	0,018	5,77250625	103,9051125	41,562045
	7	[illegible]	[illegible]	[illegible]	[illegible]
	8	0,032	7,696675	246,2936	98,51744
	9	[illegible]	[illegible]	[illegible]	[illegible]
	10	0,05	9,62084375	481,0421875	192,416875
	11	0,0605	10,58292813	640,2671516	256,1068606
	12	0,072	11,5450125	831,2409	332,49636

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