

Master's Dissertation/
Trabajo Fin de Máster

MATHEMATICS THROUGH CLIL: A COMPREHENSIVE LITERATURE REVIEW AND A DIDACTIC PROPOSAL TO INTRODUCE CLIL IN AN ECUADORIAN MONOLINGUAL SCHOOL

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ABSTRACT

This MA dissertation focuses on the implications of teaching mathematics in CLIL by means of an extensive literature review, starting from the general features of CLIL and the context of mathematics teaching in mainstream education. Bearing in mind these general notions, an exhaustive analysis of mathematics teaching by using English as the vehicular language is carried out taking into account aspects such as the importance of the language of instruction, the development of communication skills, the role of scaffolding to achieve the expected learning outcomes, methodologies, materials and ICT, challenges, engagement, and the effects of learning mathematics through a second language.

Finally, a lesson plan is proposed to introduce CLIL in the tenth grade of CSE (corresponding to the third grade of CSE in Spain) in an Ecuadorian monolingual school, adapting CLIL to the context of the target group by using student-centred methodologies and the support of ICT.

KEYWORDS: CLIL, mathematics, Compulsory Secondary Education, Ecuadorian monolingual school, lesson plan.

RESUMEN

Este TFM estudia las implicaciones de la enseñanza de las matemáticas con AICLE mediante una extensa revisión de la literatura, partiendo de las características generales de AICLE y el contexto de la enseñanza de las matemáticas en la educación general. Teniendo en cuenta estas nociones generales, se realiza un análisis exhaustivo de la enseñanza de las matemáticas utilizando el inglés como el idioma vehicular considerando aspectos tales como la importancia del idioma de instrucción, el desarrollo de destrezas comunicativas, el papel del andamiaje para lograr los resultados de aprendizaje esperados, metodologías, materiales y TIC, desafíos, compromiso y los efectos del aprendizaje de las matemáticas a través de un segundo idioma.

Finalmente, se propone una unidad didáctica para introducir AICLE en el décimo grado de ESO (tercero de ESO en España) en una escuela monolingüe ecuatoriana, adaptando AICLE al contexto del grupo objetivo mediante metodologías centradas en el estudiante y las TIC.

PALABRAS CLAVE: AICLE, Matemáticas, Educación Secundaria Obligatoria, escuela monolingüe ecuatoriana, unidad didáctica.

1. INTRODUCTION

CLIL appears as an innovative approach which “is something more than an educational need” (Lorenzo, 2007: 27); it aims to cross language barriers and make knowledge universal. Marsh (2002: 11) considers CLIL as a solution essential to deal with a European problem and, widening this idea, as a personal view, in the future, CLIL might become almost ubiquitous in language teaching.

Given the importance of CLIL as a new educational model aspiring to improve foreign language (henceforth, FL) competences, this dissertation analyses the impact of CLIL on mathematics learning and its effectiveness comprising each aspect required in order to fulfil the CLIL criteria and the curricular objectives of the subject successfully.

To begin with, the second chapter (“Theoretical framework”) summarises the main theoretical aspects of CLIL since it is crucial to know the bases of this approach, and, subsequently, evaluate its practical application. In addition, important notions of mathematics teaching in traditional education are analysed based on a bibliographical search as an initial point to determine the curricular implications of the contents, possible difficulties, and methodologies to apply.

In the next chapter (“Mathematics teaching through English”), there is a compilation of data from an extensive literature review as a way to study the implications of teaching mathematics by means of CLIL and its effects as to content knowledge as well as language skills

Finally, in order to complement this information, a CLIL lesson plan will be proposed based on the theoretical research previously conducted. Therefore, for the first time a CLIL unit will be inserted in the tenth grade of Compulsory Secondary Education (henceforth, CSE) (equivalent to the third level of CSE in Spain) at San Felipe Neri school, an Ecuadorian monolingual public school in Riobamba with a low socio-economic status bearing in mind its specific teaching context, requirements, language skills, and the expected learning outcomes.

1.1. Justification

Currently, the world is undergoing radical and swift changes that demand to establish an endless communication network that can eliminate language barriers; thus, speaking different languages has become an urgent need to adapt to a growing globalised and technological community (Mariño Ávila, 2014: 251). A language is not

only a system of communication but also the reflect of a culture that impacts on daily lives. Nowadays, learning a second language (henceforth, L2), particularly English, has become a communication skill necessary to actively take part in a society open to multiple cultures. As Roberts, Leite & Wade (2017: 116) point out “monolingualism is the illiteracy of the twenty-first century.”

Furthermore, traditional education systems based on teaching through the mother tongue have widely contributed to such high illiteracy rates in European countries. To illustrate, according to surveys and reports performed in order to ascertain the level of English in European countries, Spain stands out for having one of the lowest levels of the language in question, which shows concerning levels of monolingualism still existing in a globalised world (cf. Zafra, 2019).

In light of this situation, CLIL appears as a decisive educational approach whose objective is to institute bilingual education as a mainstream system. Learning contents through a FL goes beyond a mere combination of both aspects; this fusion demands a deep and total understanding of all the aspects involved in CLIL such as aims, learning outcomes, methodologies, strategies, etc. In other words, the whole teaching-learning process through a L2 (Graff, 2016: xviii).

The essence of CLIL focuses on teaching non-linguistic subjects through a non-native language, whereby, learners can simultaneously acquire language and content competences in a real context. Nevertheless, this ambitious goal entails a lot of effort, competences and teaching training taking into account that accomplishing the effective learning of a subject in a FL requires concrete actions, pedagogical strategies and methodologies so that students can learn content knowledge in the same or even in a higher level than they do in their mother tongue but acquiring FL skills simultaneously.

Guaranteeing content acquisition stands as a tremendous challenge for teachers considering the implications of the vehicular language, and even more with respect to the evident complexity of teaching exact sciences such as mathematics. Being a mathematics teacher is certainly a huge responsibility that entails countless professional challenges and the struggle with complex processes (Chapman, 2012: 263); consequently, being a CLIL teacher in charge of mathematics is much more demanding.

Mathematics by its nature implies an arduous labour to carry out in the classroom at any educational level. Moreover, there are still misconceptions of how students grasp concepts: one of them is to think that everyone learns in the same way,

which leads to not achieve the expected learning results. Chinn (2016: 53) mentions that the difficulties found in mathematics learning are worldwide. Culturally, mathematics is a subject full of beliefs that increase the feeling of insecurity and fear of learning it. Furthermore, students' needs may not be taken into account, which seriously affects the achievement of the expected learning outcomes.

Bearing in mind the general difficulties of teaching the subject in question while using English as the vehicular language as well as "the controversy of bilingual education in Spain" (my own translation from Pavón Vázquez, 2018: 21) and perhaps in the rest of the world, this study is pertinent in order to delve into the analysis of the context and effectiveness of learning mathematics in bilingual education by means of a collection of data from secondary sources of information, to finally propose a CLIL lesson plan adapted to a specific learning context.

1.2. Objectives

1.2.1. General objective

To analyse the context and identify the mathematics teaching and learning needs related to a CLIL approach to finally design a CLIL lesson plan adapted to the context of San Felipe Neri school (Riobamba, Ecuador).

1.2.2. Specific objectives

- To determine the characteristics and requirements of the CLIL mathematics classroom in CSE.
- To assess the communication skills used in the CLIL mathematics classroom in CSE.
- To compare mathematics teaching in bilingual education to traditional education in CSE.
- To review methodologies and material applied in mathematics teaching through CLIL in CSE.
- To analyse the effects of learning mathematics by using English as a means of instruction in CSE.

2. THEORETICAL FRAMEWORK

2.1. Content and language integrated learning (CLIL)

CLIL has emerged as an innovative approach focused on both contents and languages working in “a joint role” (Marsh, 2002: 58) in which both factors do not compete with each other but rather complement each other, so thinking about the key to success is thinking about the learning results obtained by studying the contents and a FL synchronously (Coyle, Philip & Marsh, 2010: 1).

CLIL materialises based on Canadian and American immersion approaches before the need of possessing a European bilingual approach that counteracts the conventional monolingual system considered as “second rate education” (Lorenzo, 2007: 35). However, CLIL differs from its predecessors establishing its own underpinnings that firmly underlie a new educational pathway. As Ball, Kelly & Klegg (2015: 37) mention, it focuses on teaching non-linguistic subjects in specific schools without covering the entire curriculum. Besides, learners do not need a high language competence to take a CLIL course, they begin with a basic L2 level and along the proper learning process, their language skills flourish.

All of these aspects have made CLIL programmes evolve because they adapt to the students’ context and needs. By means of this new approach, students have the opportunity to become the constructors of their knowledge fostering learning autonomy, but, at the same time, focusing on cooperative learning.

2.1.1. Definition

Searching for the perfect definition for “CLIL” is similar to looking for a needle in a haystack. The research in terms of CLIL encompasses a really extensive literature which provides with solid foundations of what it is from different points of view and studies conducted by several authors.

Nevertheless, after comparing and interpreting countless definitions of CLIL, it is worth choosing one that reflects the essence of this approach and conveys the most comprehensible input. That is the case of the definition provided by Coyle, Philip & Marsh (2010: 1), who define CLIL as “a dual-focused educational approach in which an additional language is used for the learning and teaching of both content and language.” Moreover, it is ideal to mention that CLIL is not only about content and

language learning, but also the cultural and cognitive aspects that are fundamental parts of the educational process (Nikula, Dafouz, Moore & Smit, 2016: 1).

2.1.2. Components

CLIL is a conglomeration of striking features that make it be seen as something more than an educational approach as described by some experts such as David Marsh and Do Coyle. Nevertheless, this characterisation may lead to several interpretations; just to mention, some scholars consider CLIL as an integration of techniques that foster and favour language acquisition in the classroom (Cenoz, Genesee & Gorter, 2014: 245).

In this case, from a personal viewpoint based on a literature review, it is meaningless to categorise it only as a combination of techniques and other strategies since CLIL goes much beyond. Practically, as an approach, it covers every factor entailed in the teaching-learning process, which are identified as the 4Cs of CLIL referring to content, communication, cognition and culture (Coyle, Philip & Marsh, 2010: 41).

According to Cano Cuadrado (2013: 67), these components interrelated condition an effective practice of CLIL, besides involving learners' knowledge progress, strategies and skills to understand contents.

It is possible to make an overview of each component summarising from Bentley (2010: 7):

- Contents: related to specific curricular subjects such as art, citizenship, geography, mathematics, natural science, physical education, technology and so on.
- Communication: the vehicular language for learning in oral and written forms encouraging a major number of classroom interactions.
- Cognition: thinking skills (creativity, reasoning, critical thinking, etc.) for studying subjects.
- Culture: learners must understand themselves, their context and their and social/cultural backdrop (behaviours, mindfulness, values, etc.) (cf. Bentley, 2010: 7).

2.1.3. Advantages and disadvantages

Learning a subject through a FL entails a cognitive challenge for both teachers and learners, and similarly to other approaches, there are some assets and drawbacks that should be discussed briefly.

Coyle, Philip & Marsh (2010: 10) highlight “a learner’s cognitive development” as the main advantage of CLIL whereby students can acquire an ability to think in other languages regardless of their extent. However, this goal can be attained as long as teachers make methods more suitable to teach contents in a non-native language.

CLIL, due to its duality, not only fosters linguistic competences, but favours cultural development and strengthens cognitive processes as well (Coyle, Philip & Marsh. 2010: 10). CLIL assets are undeniable since it benefits both teachers and students.

Šulistová (2013: 50) details some favourable aspects of using CLIL, demonstrated in the improvement of students’ critical thinking, the acquisition of significant learning by means of real life situations, cooperative learning and teamwork. Moreover, CLIL fosters cultural awareness and employability.

On the other hand, it is pertinent to analyse the disadvantages that may affect the teaching-learning process and that should be considered with the purpose of looking for strategies in order to minimise their impact or even prevent them.

To begin with, Šulistová (2013: 50) similarly mentions some negative facts of CLIL incidence, which are mainly focused on four aspects such as time consumption, level of FL required from both students and teachers, unfamiliarity with CLIL theoretical underpinnings, and insufficient teacher training courses.

These aspects might be some of the reasons for negative criticism against this approach since it is a fact that, if those factors are not controlled, CLIL will not reach its maximum potential. Time consumption, for instance, is a problem to deal with in order to execute CLIL projects successfully: “Many teachers find neither the time nor the inspiration to undertake” the design of CLIL materials, and doing it certainly “requires time and attention” (Ball, Kelly & Clegg, 2015: 173).

Equally, the lack of training and mastery of CLIL theory undoubtedly hinder teacher labour and, consequently, CLIL effectiveness. Pérez Cañado (2018: 213) states that the mastery of scientific knowledge impacts on both the contents to be taught and the theoretical foundations of CLIL that need to be widely understood by CLIL practioners to perform an effective role in the classroom.

In conclusion, it is possible to say that the disadvantages of CLIL might not be in teaching-learning methodologies, but rather the negative aspect could be the complexity of the requirements and competences necessary to successfully perform in CLIL, which unfortunately have not been overcome yet.

2.1.4. Development of CLIL in Europe

Education through a FL is not a current tendency. The need for education and communication stems from ancient civilizations, and a clear example of that is Roman education by using Greek as the vehicular language in order to promote language acquisition as well as social and professional opportunities (Coyle, Philip & Marsh, 2010: 2). Currently, the need for an effective education and communication in FLs still prevail. In Europe, integrative purposes underlie the urgency of speaking English, motivated by the desire of an internationalised and globalised society (Devos, 2016: 18).

The objective of speaking a FL is linked to the need of adapting to a changing world that requires professionals with more competences that are not only focused on the field of study, but on new paradigms of social and cultural integration as well. Learning languages is currently associated to globalisation, economy and the ongoing need of social integration among nations (Coyle, Philip & Marsh, 2010: 2). This need of flourishing in Europe forced to seek a new bilingual approach as effective and efficient as Immersion Approaches but with the European distinction.

For the European Commission, FL learning favours economy, interculturality and mobility; as a result, the implementation of European educational programmes (for example, Life-long Learning) and the direct funding for CLIL projects are strategies to develop the mother tongue and learn FLs (Devos, 2016: 18-19).

According to Wolff (2002: 47), CLIL as a pedagogical concept has been present in European schools for more than 30 years underscoring the relevance of CLIL to accomplish language goals in accordance with the European language policy. Additionally, Frigols Martín (2008: 221-222) notes that this approach has gained great visibility; therefore, it is being applied in most of the European countries. Spain stands out due to its growing interest that has led CLIL programmes to be supported as part of the public educational policy and implemented in mainstream schools.

2.1.5. Factors to succeed in CLIL

Teaching contents in an additional language could involve much more situations to consider before implementing an educational programme than in conventional approaches. CLIL encompasses content-based instruction and language teaching, but without putting more pressure on one or the other; this aspect being the main difference from common language approaches (Coyle, Hood & Marsh. 2010: 1). Language learning becomes language acquisition when it occurs in a real context with continuous content-based interactions associated with students' culture and, simultaneously, developing cognitive skills.

Mehisto (2008: 93) remarks on the need for stakeholders to be aware of the complexity involved in managing the educational changes generated by CLIL. He also refers to the lack of awareness of the objectives of this approach, which complicates the implementation of CLIL programmes.

Besides, CLIL may be considered as a complex process to carry out because of its requirements to execute an effective programme; thus, it is crucial to bear in mind the following criteria abbreviated from Ball, Kelly & Clegg (2015: 10-19):

- Learners need a good level of command in the vehicular language; however, it is possible to teach students with low levels of the target language and keep an acceptable level of content knowledge on condition that effective CLIL pedagogy is applied.
- L2 exposure is paramount given that the more exposed to social situations learners are, the more language skills they develop.
- L1 literacy and cognitive skills play an important role to make learning through a FL easier.
- The socio-economic background is not determining to successfully perform in CLIL; nonetheless, it is needed to consider that those students with a low social status could require further support.
- Teachers must master both language and contents, in addition to strong teaching skills focused on bilingual education.
- CLIL resources are a decisive factor for learners in order to have available varied didactic materials with contents in an appropriate language level. These materials have to be in accordance with subjects,

students' linguistic and cognitive skills, learning outcomes, CLIL dimensions and motivation.

- Collaboration between language and content teachers is crucial from the beginning in order to work on the same teaching objectives, which should lead to a good management of content and language planning as a whole.

2.2. Mathematics in mainstream education

Knowledge is not a new thing, and neither is learning. Education has been an evolving process that has been changing according to emerging needs; thus, the transformation of education has made multiple teaching paradigms appear based on how students learn (Cowan, 2006: 14). Mathematics learning may become a complex process if students' learning styles and needs are not taken into account before planning a lesson.

A recurrent mistake is to believe that all students learn at the same time and under the same conditions, which could be the reason why some students feel frustration and aversion to mathematics. Students' dissatisfaction stands for a real problem that needs to be solved urgently, recalling that this subject is present in every daily aspect. Edwards (2007: 2) believes that mathematics is closely linked to the daily context in which people operate; therefore, mathematical operations have a reason to be and to exist based on the needs that arise even in simple everyday situations. The context necessary to learn mathematics is life in its entirety.

Mathematics is part of the group of core subject matters that must be learnt from primary education until higher education as per the curriculum of each country. Hiebert & Carpenter (1992: 67) state that this subject can be clearly understood provided that maths contents are mentally depicted as part of a system of representations. In other words, mathematics cannot be taught in isolation but supported by numerous connections that foster a better understanding in context.

François & Van Bendegem (2007: 17) express the importance of significant learning in mathematics teaching, so that learners can understand the practical use of the subject and the commonalities with everyday situations. Maths contents should be seen as something positive for life and not only as a compulsory subject to be passed by following difficult procedures and memorising formulas. The instruction context and the connections linked to the subject make its learning easier focusing on real aspects

due to the fact that “the primacy of mathematics comes from the fact that it shows the extremely non-obvious relationships among things” (Dimitric, 2016: 4).

2.2.1. Mathematics curriculum

The Ecuadorian Ministry of Education defines a curriculum as an expression of the educational project that the members of a country or a nation prepare in order to promote the development and socialisation of new generations and in general of all its members (my own translation; extracted from *Ministerio de Educación Ecuatoriano*, 2016: 4). The curriculum of a country reflects a series of guidelines, objectives, skills to be developed as well as evaluation criteria that guide the teaching-learning process of the compulsory curricular subjects at different educational levels.

Mathematics may be seen as one of the most complex subjects, but at the same time, it is one of the most important ones, given that “mathematics has always been a major component of the school curriculum” (Robitaille & Dirks, 1982: 3) in each country. This subject is part of daily life with an essential value that is greater than any science or academic discipline (Kilpatrick, Hoyles & Skovsmose, 2005: 19).

In accordance with the Ecuadorian educational legislation, the Ecuadorian curriculum (which is going to be analysed in this dissertation) details the different curricular notions for the levels of General Basic Education (primary and secondary education) and General Unified Baccalaureate. In this particular case, the former will be the object of study in order to examine the mathematics curriculum for the eight, nine and tenth grades of CSE (equivalent to the first, second and third grades of CSE in Spain). After passing the tenth grade, Ecuadorian students gain admission to baccalaureate studies.

The Ecuadorian mathematics curriculum for CSE (named as sublevel of *Educación Básica Superior* in Ecuador), establishes the importance of learning three content blocks, which are algebra, geometry, and statistics (*Ministerio de Educación Ecuatoriano*, 2016: 224-227). Based on those mathematical branches, there are seven general curricular objectives for CSE, which have to be reached after three academic years at this educational sublevel. For a better understanding, it is important to clarify that in Ecuador, the curricular objectives are coded as follows:

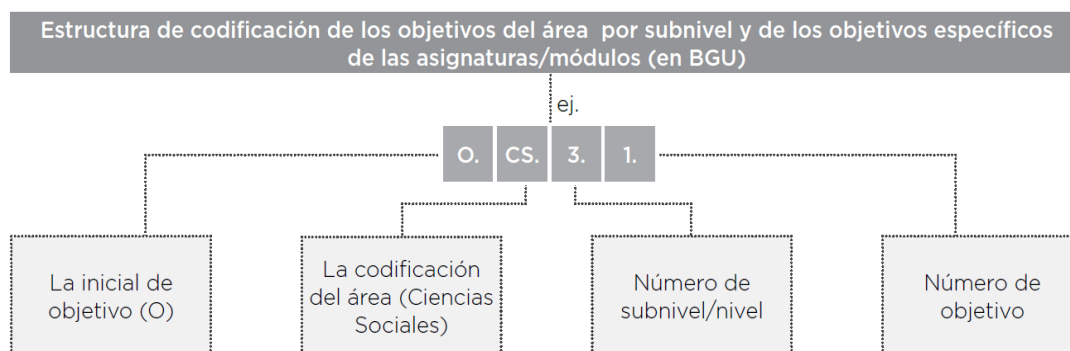


Figure 1: Codification structure of the objectives of each subject. Extracted from *Currículo de los niveles de educación obligatoria* (Ministerio de Educación Ecuatoriano, 2016: 45)

In the case of mathematics, the letter M is used as representation of the subject, and the number 4 corresponds to the sublevel of *Educación Básica Superior* (CSE). Bearing in mind these details, the curricular objectives for mathematics in Ecuadorian CSE are detailed below.

O.M.4.1.	Reconocer las relaciones existentes entre los conjuntos de números enteros, racionales, irracionales y reales; ordenar estos números y operar con ellos para lograr una mejor comprensión de procesos algebraicos y de las funciones (discretas y continuas); y fomentar el pensamiento lógico y creativo.
O.M.4.2.	Reconocer y aplicar las propiedades conmutativa, asociativa y distributiva; las cuatro operaciones básicas; y la potenciación y radicación para la simplificación de polinomios, a través de la resolución de problemas.
O.M.4.3.	Representar y resolver de manera gráfica (utilizando las TIC) y analítica ecuaciones e inecuaciones con una variable; ecuaciones de segundo grado con una variable; y sistemas de dos ecuaciones lineales con dos incógnitas, para aplicarlos en la solución de situaciones concretas.
O.M.4.4.	Aplicar las operaciones básicas, la radicación y la potenciación en la resolución de problemas con números enteros, racionales, irracionales y reales, para desarrollar el pensamiento lógico y crítico.
O.M.4.5.	Aplicar el teorema de Pitágoras para deducir y entender las relaciones trigonométricas (utilizando las TIC) y las fórmulas usadas en el cálculo de perímetros, áreas, volúmenes, ángulos de cuerpos y figuras geométricas, con el propósito de resolver problemas. Argumentar con lógica los procesos empleados para alcanzar un mejor entendimiento del entorno cultural, social y natural; y fomentar y fortalecer la apropiación y cuidado de los bienes patrimoniales del país.
O.M.4.6.	Aplicar las conversiones de unidades de medida del SI y de otros sistemas en la resolución de problemas que involucren perímetro y área de figuras planas, áreas y volúmenes de cuerpos geométricos, así como diferentes situaciones cotidianas que impliquen medición, comparación, cálculo y equivalencia entre unidades.
O.M.4.7.	Representar, analizar e interpretar datos estadísticos y situaciones probabilísticas con el uso de las TIC, para conocer y comprender mejor el entorno social y económico, con pensamiento crítico y reflexivo.

Table 1: Curricular objectives for Mathematics in Secondary Education. Extracted from *Currículo de los niveles de educación obligatoria* (Ministerio de Educación Ecuatoriano, 2016: 879)

To reach those objectives, the curriculum details a group of skills with performance criteria that have to be developed with the support of ICT throughout the three grades under the guidelines of the evaluation criteria and indicators, specifically focused on the following maths contents:

ALGEBRA AND FUNCTIONS	GEOMETRY AND MEASURES	PROBABILITY AND STATISTICS
Real numbers Equations Inequalities Functions	Geometric shapes Pythagoras theorem Trigonometric functions	Data collection Measures of central tendency Deviation and variance

Table 2: Mathematical contents for Ecuadorian Compulsory Secondary Education. Following *Currículo de los niveles de educación obligatoria* (Ministerio de Educación Ecuatoriano, 2016: 224-227).

At the end of each grade, Ecuadorian students are expected to demonstrate the mathematical abilities and knowledge detailed in the curriculum by passing formative and summative assessment with a final grade of at least 7/10. After finishing the three grades of CSE, Ecuadorian students are promoted to the first grade of baccalaureate (*Bachillerato General Unificado*), that is the counterpart of Non-compulsory Secondary Education (henceforth, NCSE) in Spain. In Ecuador, baccalaureate education is compulsory and lasts for three academic years, so students are supposed to finish it when they are 18 years old.

2.2.2. Pedagogical components

Mathematics teaching goes beyond only mastering mathematical content knowledge; it also entails a deep understanding of how to teach through the application of numerous pedagogical factors that guide teaching work in accordance with the curriculum. Fennema and Franke (1992; as cited by An, Kulm and Wu, 2004: 169) state that the mathematics teachers' knowledge should not be composed only of content knowledge and mathematical representations. They need to get to know their students (cognition) and be able to develop other skills such as decision making and problem solving.

Correspondingly, summarised from An, Kulm & Wu (2004: 146-147), pedagogical knowledge content can be defined as the essential knowledge that turns out to be paramount to accomplish effective teaching involving three main factors such as content knowledge (mathematics knowledge), knowledge of curriculum (curricular

notions, objectives, skills, indicators, etc.) and teaching knowledge (students' thinking, motivation, instruction, methodologies, etc.).

These foundations confirm that teachers guide the teaching-learning process, whereby they must have both content and pedagogical strong competences in order to conduct this process effectively and successfully. Furthermore, another key aspect is to provide students with a holistic view and a real connection with other disciplines. Ruiz, Alfaro Carvajal & Gamboa Araya (2003: 285) mention that one of the essential points in mathematics teaching is to know how to design a lesson in order to produce significant learning. It is determining to strengthen reasoning not only related to science and technological implications, but also dealing with the different panoramas that can be present in life (Anthony & Walshaw, 2009: 245).

The ways mathematics is taught underlie students' learning and engagement; thus, regardless of any approaches managed in mathematics teaching, Anthony & Walshaw (2009: 148) designed ten principles to guarantee an effective pedagogy to teach mathematics by encompassing several factors such as promoting learning communities and constructive discourse classroom, tasks to intensify students' thinking, and the importance of teachers' knowledge.

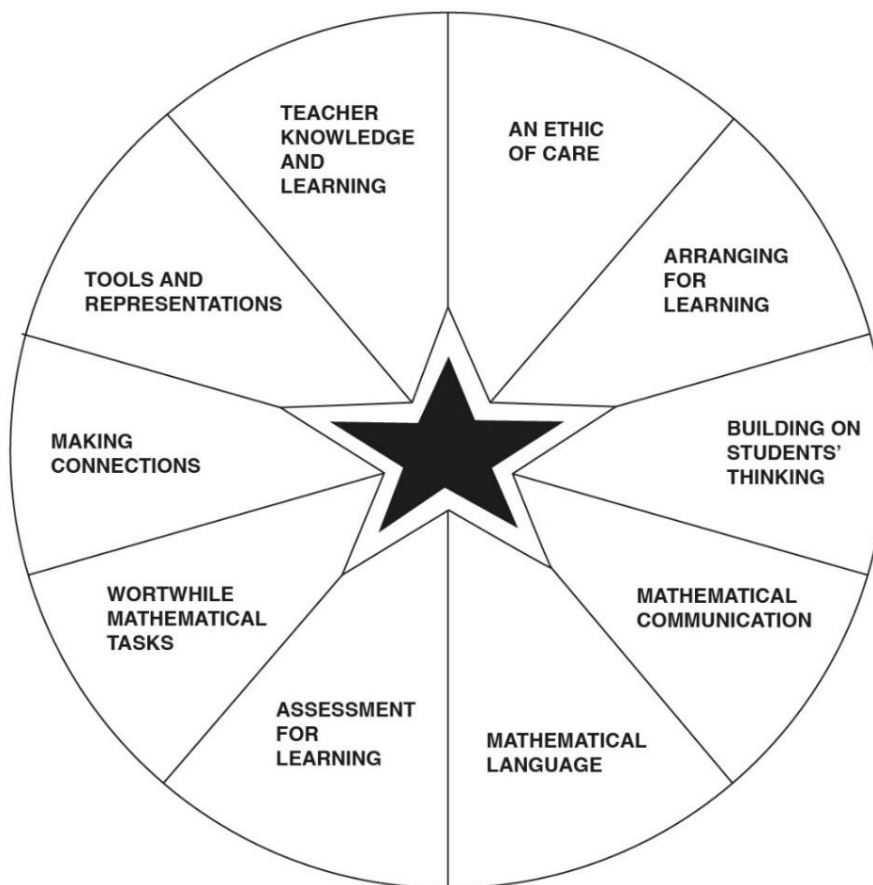


Figure 2: Principles of effective pedagogy of mathematics (Anthony & Walshaw 2009: 148).

The authors of these principles consider them as all the requirements for an effective teaching class, where students take on the main role, but always under teachers' support. Hence, a maths teacher is responsible for creating a community in the classroom to reach mathematical goals based on identity and proficiency, giving opportunities to work both independently and collaboratively, building learning based on interests and experiences with real associations with daily life, assessing but supporting, fostering dialogue with appropriate terms and communication, and finally, conducting each class with proper activities and tasks and using tools and resources that respond to students' needs (cf. Anthony & Walshaw, 2009: 149-159).

2.2.3. Teaching methodologies

Mathematics intervenes in a wide number of aspects thanks to its transversal knowledge, procedures, and skills. It is actually a complete tool that provides children and adults with the opportunity to interact in a variety of contexts, therefore being able to organize, manipulate, and communicate information (Government of Ireland, 1999: 2). Barmby, Bilsborough & Harries (2009: 2) describe mathematics understanding as a system of cognitive representations that provide knowledge with meaning and strong connections, denoting that the more closely mathematical procedures are linked to real contexts, the better mathematics is understood.

Given that mathematics is a driving force in daily, scientific, and technological aspects, the use of proper methodologies to teach maths contents influences the learning rate achieved. In order to attain the curricular learning outcomes, teachers have to be aware of the different methodologies applied in the classroom, some based on traditional paradigms that are still applied, whereas others focus on innovative approaches (Lessani, Suraya & Abu Bakar, 2017: 1286).

2.2.3.1. Traditional method

This method is based on traditional education (teacher-centred), where a teacher is responsible for giving lectures while students pay attention to take notes, and the class ends with the teacher sending homework, in consequence, involving them in a monotonous process. The teacher is the lynchpin to transmit knowledge, while students are simple recipients of the message by only applying receptive skills. (Noreen & Rana, 2019: 149).

2.2.3.2. *Problem solving*

Problem solving is a structured method to solve maths problems by following four stages: understanding the problem (to read and understand the information given), devising a plan (to select proper strategies to find a solution), executing the plan (to apply the strategies), and looking back (to analyse the solution and the answer). By means of this framework, students may also use different methods to learn mathematics concepts and develop skills enriched with meaning and connections. However, this approach could promote too methodical and linear thinking, preventing students from using their creativity to learn (Ortiz¹, 2016: 4, 9).

2.2.3.3. *Cooperative learning*

Cooperative learning is a student-centred methodology whereby pupils work in groups to achieve the successful completion of learning tasks. Students need to engage with the objective of the task by working as a whole and contributing with different ideas and skills. (Leikin & Zaslavsky, 1999: 240).

Zakaira, Chin & Daud (2010: 274) conducted a study to determine how cooperative learning impacts on mathematical achievement and learners' attitude towards the subject. From this research, they found that through cooperative learning, students demonstrated higher academic achievement and engagement than through traditional approaches, due to the fact that cooperative learning opens several opportunities so that students can actively participate, generate knowledge and propose solutions through discussions, collaboration, creativity and problem solving.

2.2.3.4. *Discovery learning*

Discovery learning promotes the construction of knowledge through questions. This aspect is related to constructivism and new educational models that allow students to actively work in schools (Hanna *et al.*, 2012: 1009). It is applied to solve maths problems, so students can focus previous experiences and knowledge on discovering new facts or solutions.

Mathematics is not an isolated discipline; quite on the contrary, it is a direct connection to several academic and life aspects whereby learners can move through the generation of mathematical depictions and ideas (Gardiner, 2016: 9). The main

¹ In the case of Spanish-speaking authors, the second surname is always provided when it is available.

problem is the fact that students only tend to memorise concepts and theories, which prevents them from developing new skills and building their own knowledge.

Through discovery learning, mathematics is learnt by doing and developing several skills such as problem solving, creativity, reasoning, critical thinking, analysing, modelling and so on. The teacher is responsible for guiding and facilitating students to construct their knowledge by applying scientific knowledge in problems and projects encouraging students to investigate and go into detail about the topic (Kistian, Armanto & Sudrajat, 2017: 1).

2.2.4. Student motivation/frustration

Students are naturally curious and intuitively motivated to learn something new every day. Sorensen (2006: 3) claims that every student is motivated; nonetheless, students are not motivated to do what teachers want. As learners grow up, they have more school responsibilities and must assimilate a lot of information from different subjects, which can be extremely exhausting and frustrating if the teaching process is not appropriately carried out. Teaching achievement is learning achievement that is determined by numerous factors, among which, motivation stands out as one of the pivotal aspects that conditions mathematics learning (Fuqoha, Budiyo & Indriati, 2018: 203).

Despite the fact that mathematics teaching is not something new and is present at all educational levels, thinking of an effective way of teaching mathematics can lead to an endless and heated discussion (Abramovich, Grinshpan & Milligan, 2019: 2). Mathematics is a subject that demands a huge cognitive effort and, as a result, motivating students to learn it can sometimes be an arduous process when teachers do not apply proper methodologies to engage students and give them the opportunities to participate actively.

Nowadays, teacher-centred approaches are still used in mathematics teaching, which has brought out several behaviours and opinions on the subject. Hoyles, Kilpatrick & Skovsmose (2005: 9) point out that some students are reluctant to do maths homework, whereas others enjoy maths activities such as discussions or problem solving. Some students may believe that mathematics is extremely difficult or even useless, or they may simply feel disappointed and unable to learn because of some weaknesses.

Education never stops and, undoubtedly, mathematics education is a continual process that aims students to understand systematically and cognitively all the procedures, concepts and mathematical abstractions (Lessani, Suraya & Abu Bakar, 2017: 1287); on the contrary, students may be frustrated, especially, when they have to solve maths problems. Frustration is a negative emotion due to encountering a factor or situation that hinders the satisfaction of a need or the accomplishment of a goal, affecting the successful completion of a task (Sierpiska, 2006: 121).

Teachers must facilitate maths understanding in a natural context showing that mathematics can be an enjoyable process by using appropriate methodologies, strategies and resources. Moreover, mathematics teachers should provide students with confidence and motivation by encouraging them to delve into math problems through critical thinking, decision making, and creativity as if they were their own everyday problems (Lessani, Suraya & Abu Bakar, 2017: 1287).

2.2.5. Difficulties to teach mathematics

Problems with mathematics learning have become worldwide and a real challenge for teachers to overcome (Chinn, 2016: 53): “Teaching maths is a complex and demanding process” (Kurnik, 2008: 419) that requires teachers to be well qualified and trained as to teaching strategies and student-centred methodologies that allow students to take part in the construction of knowledge.

Given the importance of mathematics in numerous fields; low academic performance represents a concerning problem, which could be because students tend to consider mathematics as an extremely difficult and even boring subject, thus, lacking a real interest in approaching it (Zakaira, Chin & Daud, 2010: 274).

It is possible to realise that one of the biggest challenges to face in mathematics teaching is to deal with the perception of difficulty and boredom that students may have towards maths contents; which can cause frustration and low academic achievement; accordingly, teachers seeing poor learning outcomes may experience dissatisfaction and lack of confidence. Keefe (1997, as cited by Zakaira, Chin & Daud, 2010: 274) points out that frustration among teachers and students has a strong influence on attaining outstanding learning outcomes in mathematics, then this problem has to be overcome.

In conclusion, maths teachers have to apply teaching methodologies that enable students to see mathematics learning as an opportunity of experiencing and

finding and not only as a compulsory subject. Every class should be dynamic and pleasant in order to encourage students to become lifelong learners.

3. MATHEMATICS TEACHING THROUGH ENGLISH

Mathematics teaching in mainstream education is itself a real challenge, and even more, having to do it by using English as the vehicular language. This last aspect refers to CLIL, an approach that centres on content teaching through the simultaneous development of linguistic competences. While CLIL implies teaching non-linguistic subjects in a FL, this fact does not mean that English mandatorily has to be the target language of teaching. Nevertheless, according to Azman & Shin, (2012: 109), as English is a language of utmost importance throughout the world, most people aim at a higher command of English to establish effective communication everywhere.

English is the preferred FL used in CLIL (Jäppinen, 2005: 150); by contrast, mathematics might not be the favourite subject taught in CLIL due to the nature of its contents unlike other disciplines such as arts, biology, geography, history or music.

The imposition of learning mathematics in an additional language could turn out to be intimidating for students taking into account that the language of instruction conditions the understanding of contents at all educational levels (Ajayi & Lawani, 2015: 304). Hence, students should receive constant linguistic support to not only boost the linguistic competence, but also guarantee a complete understanding of the mathematical content since “CLIL is a dual-focused approach” (Sabet & Sade, 2012: 89).

3.1. Overview of CLIL in science

The objective of CLIL is to promote a realistic and significant learning through the use of authentic language in the classroom by means of social and academic interactions and the implementation of didactic materials and resources that support language acquisition (Gabillon & Ailincăi, 2013: 169). Aragón Méndez (2007: 155) points out that bilingual education programmes mainly focused on experimental science favour FL learning, owing to the fact that science provides technological and scientific knowledge that helps understand the behaviour of a globalised society.

Given the importance of science in daily life, in order to implement CLIL in a science classroom, language has to be adapted to a scientific context, but without forgetting associations with students' background and culture (Prieto Ruz, España

Ramos & Martín Gámez, 2012: 72). Before beginning a class, it is pivotal to establish connections between students' schemata and the contents to be reviewed.

In CLIL, content and language go hand in hand since content learning drives to language acquisition. The immersion of CLIL in science may involve more complexities and factors to bear in mind than in humanistic subjects; some of these factors are the comprehension of scientific procedures and methods, the accuracy of specialised vocabulary and grammatical structures, and the connection between science, life and the environment with which people relate (University of Cambridge, 2011: 3).

The connection between language and science is undeniable (Aragón Méndez, 2007: 156); therefore, science teachers immersed in CLIL programmes must be well qualified demonstrating mastery of both scientific contents and the academic language that learners need in order to establish an effective communication in scientific contexts (Cimermanová, 2017: 5; University of Cambridge, 2011: 3).

3.2. CLIL in mathematics

Even though, there has been an evident trend towards CLIL in the humanities like social science (Viebrock, 2009: 64); mathematics is one of the non-linguistic subjects that can be taught through CLIL approach, which logically entails teaching calculations, formulas, theorems, and other cognitively demanding procedures through a FL used as the main communication vehicle in the learning environment. Given this dual focus, the expected learning outcomes centre on learning content knowledge and acquiring linguistic competences in the target language, but without affecting the mother tongue. On the contrary, the first language (henceforth, L1) should benefit from this dual-focused teaching process.

Mathematics is made up of abstract concepts that even in L1 may be difficult to understand, then the question is if it is possible to express these concepts in more than a language with the same level of comprehension as developing language skills in L2 (Viebrock, 2009: 65). This uncertainty prompts to analyse how mathematics and English as a Foreign Language (henceforth, EFL) settle into CLIL in order to attain an effective assimilation of contents plus the acquisition of the target language.

3.2.1. Relationship between mathematics and language

If learning a FL is in fact a hard task, studying core subjects in a FL is even more difficult (Kasapoğlu-Akyol, 2010: 227). Changing the language of instruction entails

ensuring that content subject benefits from it (Prochazkova, 2013: 24), and specifically regarding maths contents, it is essential to manage a clear perspective on how to efficiently integrate language and content. Needless to say, in CLIL mathematics, teachers should demonstrate at least competent language skills in order to communicate in both conversational and academic contexts, considering the abstraction and complexity of mathematical concepts and the language support that students need.

Khalid & Tengah (2007: 1) indicate the importance of glancing at the role of language in mathematics, given that this subject has been widely considered as an independent subject supported by mechanical and rigid procedures with no creative processes, which can be said it is completely false. Mathematics, like other subjects, have strong connections with each other and with real-life contexts whereby maths contents should be part of a transdisciplinary teaching-learning process that makes students understand the importance of the knowledge acquired. Certainly, the language of instruction underlies the real understanding of the content, and even more so when it is complex and cognitively demanding.

Mathematics is a language in essence due to the presence of symbols and characters that create a language that has to be decoded by learners in order to get an absolute or relative understanding of the topics (Viebrock, 2009: 66). However, most of the times, students' unwillingness to learn hampers this decoding, doing it intuitively and without a real understanding of the concepts and, given that in CLIL, learning is by means of a L2 (English, in this specific case of analysis), more cognitive skills than in L1 are required. As a result, this approach has to provide "a different perspective in Mathematics, allowing for deeper insight and understanding" (Prochazkova, 2013: 24) and engaging students.

Learning mathematics in L2 can be a cognitive challenge; nonetheless, using a different language as the means of communication in the classroom provides students with a practical perspective on contents and language, creating opportunities to learn vocabulary linked to academic and cultural contexts, and developing language skills in terms of abstraction, mathematical procedures, rationality, calculations, systematization, etc. Moreover, the use of a different language of instruction requires the application of active methodologies to boost an ongoing effective and active approach that provides a better understanding (Prochazkova, 2013: 24).

3.2.2. Communication skills

In EFL, communication skills are seen as the burgeoning of listening, reading, writing and speaking abilities to communicate in different contexts, but in the case of mathematics teaching in CLIL, it is necessary to go in depth in the communicative skills described by Cummins (1979: 198), namely Basic Interpersonal Communicative Skills (BICS) and Cognitive Advanced Language Proficiency (CALP).

BICS refer to daily and familiar language acquired in order to communicate in conversational contexts; so learners demonstrate these skills in social interactions in or out of school. On the other hand, CALP are the language skills needed to perform successfully in academic contexts. CALP development begins in the early stages of school, and these skills are essential for students to advance to the next grades. (Cummins, 2008: 2-3).

Bentley (2010: 8) comments that Jim Cummins and other academics establish a minimum period of five years to successfully develop and attain CALP, considering the cognitive effort required to perform effectively in academic settings.

Developing CALP through mathematics is still more demanding and challenging for both teachers and students because of various factors specific to the subject; for instance, maths contents comprise several formulas that are generally learnt by memorising in L1, which sometimes causes misunderstanding and oversight. Therefore, CLIL must encourage students to thoroughly analyse these formulas and their labels and names in the new language of instruction to attempt a deeper content understanding and, simultaneously, academic vocabulary learning (Prochazkova, 2013: 24). Furthermore, mathematics teaching and learning encompass both productive and receptive skills (Novotná & Hofmannová, 2000: 228).

3.2.3. Scaffolding techniques

Knowledge is a construction that requires learner's effort and eagerness as well as teacher's guidance and constant support at each stage of teaching, and this last point refers to the term scaffolding that turns out to be a pivotal concept in language teaching. Gibbons (2015: 16) defines scaffolding as a temporary structure essential to construct a solid building. From this conceptualisation, scaffolding can be seen as the foundation of a building, so the stronger the scaffolding is, the more solid the building will be.

Metaphorically, this idea of a building construction can be compared to the process of language learning, in which, scaffolding has a major role because it assists students so that learning takes place. Scaffolding is temporary assistance in the teaching-learning process based on continual interactions among teachers and peers; this process leads students to move forward to the successful completion of learning tasks (Dafouz, Linares & Morton, 2010: 13; Díaz Maggioli, 2013: 138; Gibbons, 2015: 16).

Scaffolding is crucial in CLIL and it always has to be a step ahead of students' requirements and needs. Likewise, scaffolding as a way to support students' knowledge in L2 cannot be improvised or unplanned (Coelho, 2017: 110). Every scaffolding technique has to be adapted to the needs previously detected in the target group.

The application of productive scaffolding strategies demands teachers to identify learning needs and potential in order to determine what each student can accomplish both independently and with guidance (Bikmaz *et al.*, 2010: 26). Regarding mathematics teaching through EFL, scaffolding techniques focus on the language skills needed to communicate and understand maths contents successfully.

Freiberger (2015: 5-7) describes some scaffolding strategies that, associated with effective lesson planning, may help students achieve a higher academic performance.

- Modelling: teachers demonstrate the language that students need by means of clear examples, actions and communicative behaviours that incite learners to participate in active learning tasks.
- Vital vocabulary: the introduction of significant vocabulary prior to content review is pragmatic through the use of graphic elements and representations (pictures, flashcards, posters, etc.) of the terms required. Moreover, providing vocabulary in context and the use of synonyms also facilitate the grasp of concepts.
- Visual aids and realia: the more context teachers provide, the better understanding students will gain; therefore, it is important to use all kinds of visual resources (charts, mind maps, Venn diagrams, etc.) to synthesise information and enrich vocabulary. In the same way, the utilisation of realia and visual imagery enhances the learning process.

- Suggestion of sentence starters: some students are generally nervous about expressing themselves in a different language or do not even know how to start sentences to communicate their ideas, causing them diffidence. In order to avoid these situations, teachers can suggest sentence starters to encourage students to continue with their ideas, since sometimes, learners only need a push.
- Prior knowledge activation: given that learning should be linked to experiences, an important fact is to activate schemata by using materials and resources that allow students to connect contents to their lives. Another strategy is to have friendly conversations before starting a lesson to help students create connections with what they have already learnt, seen, or experienced.
- ICT: the introduction of technology in the classroom may have a beneficial effect on promoting vocabulary or activating prior knowledge through virtual resources (games, videos, blogs, etc.). Besides, computer-based instruction develops positive attitudes and boosts a more dynamic learning environment.

3.2.4. Complexity of contents

Due to the content-language integration of CLIL, this approach stands for a complex challenge to carry out and, even more, if those contents are difficult to teach and learn in the mother tongue. Favilli, Maffei & Peroni (2013: 377-378) note that mathematics in L2 requires a great ability in standard mathematical terminology to interact in the classroom along with non-formal language skills to favour teachers' explanation and, subsequently, students' understanding.

Maths contents in CSE comprise numbers (integers, rational numbers, real numbers), algebra (algebraic interpretation, polynomials, equations, and inequalities), geometry and measures (drawing, measuring, perimeter, area, and volume, constructions, conventions, and derivations), and probability and statistics (theoretical probability, frequency, distribution, measures of central tendency, etc.) (Gardiner, 2016: 77-270; *Ministerio de Educación Ecuatoriano*, 2016: 224-227); these contents entail countless specialised terms and procedures that can be in fact hard to understand in any language; thus, changing the language of instruction may increase the difficulty level of this subject.

In order to overcome some language conflicts and the characteristic complexity of mathematics, students should engage with subject-specific language as many times as possible by revisiting essential concepts for a better understanding. Teachers have to provide students with different tasks that require the use of the same concepts over and over whereby students will become familiar with the content vocabulary and its mathematical applications and procedures (University of Cambridge, 2011: 9).

3.2.5. Teaching methodologies

CLIL promotes student-centred methodologies, setting students as the builders of their own knowledge. As a matter of fact, Van Kampen, Admiraal & Berry (2018: 224) state that CLIL essence is student-centred fostering learners' autonomy; nevertheless, learners have to receive constant support from teachers in terms of language and contents.

In mathematics education, the active involvement of students and the employment of methodologies based on learning by doing is paramount for an effective mathematics learning. By means of these methodologies, students can acquire content knowledge and develop linguistic competences through both autonomy and cooperative work.

3.2.5.1. Cooperative/collaborative learning

As explained in section 2.2.3.3, cooperative learning benefits mathematics learning in mainstream education; similarly, cooperative and collaborative learning are of utmost importance in mathematics education through a L2. Oxford (1997: 443) mentions that cooperative and collaborative learning and interaction are widely applied in mathematics teaching.

Cooperative and collaborative learning go much beyond only teamwork, but these concepts may frequently be confused as the same and, in spite of the fact that both approaches have some commonalities (Matthews, *et al.*, 1995: 35), there are some intrinsic differences that are detailed below.

Aspects	Strand 1: Cooperative Learning	Strand 2: Collaborative Learning	Strand 3: Interaction
Purpose	Enhances cognitive and social skills via a set of known techniques	Acculturates learners into knowledge communities	Allows learners to communicate with others in numerous ways
Degree of Structure	High	Variable	Variable
Relationships	Individual is accountable to the group and vice versa; teacher facilitates, but group is primary	Learner engages with “more capable others” (teachers, advanced peers, etc.), who provide assistance and guidance	Learners, teachers, and others engage with each other in meaningful ways
Prescriptiveness of Activities	High	Low	Variable
Key Terms	Positive interdependence, accountability teamwork, roles, cooperative learning structures	Zone of proximal development, cognitive apprenticeship, acculturation, scaffolding, situated cognition, reflective inquiry, epistemology	Interaction-producing tasks, willingness to interact, learning styles, group dynamics, stages of group life, physical environments

Table 3: Comparison among cooperative learning, collaborative learning and interaction (Oxford, 1997: 444).

Panitz (1999: 5) says that cooperative learning is a process whereby learners interact in order to accomplish common learning goals and is teacher-controlled, whereas collaborative learning is student-centred given that students design their own interactions keeping the control during the entire learning process.

Definitely, both approaches can be successfully integrated in mathematics learning in L2, because they allow students to engage in real interactions that do not only foster the dissemination of knowledge, but also the development of language skills.

3.2.5.2. *Task-based learning*

Task-based learning (henceforth, TBL) is a methodology focused on language acquisition through the development of tasks, although language is not the target but the task itself (Bilsborough, 2013). By doing different tasks related to specific content, students can take part in situations connected to the real world. This approach offers several learning facilities so that learners can be engrossed in the teaching-learning process by means of several strategies and useful resources to accomplish an objective or conclude an assignment (Hashemi, Azizinezhad & Darvishi, 2012: 526).

The integration of TBL and CLIL can lead to enhance “students’ conceptual understanding and improve their affective attitudes towards the subject” (Teppo, 2009: 1). By developing tasks, students can have more possibilities to revisit specialised terminology and understand its applications. According to the University of Cambridge (2011: 7), CLIL encourages TBL and offers students the opportunity to show what they know and can do.

3.2.5.3. Project-based learning

Project-based learning (henceforth, PBL) is a student-centred methodology (similar to TBL) that engages students in learning by doing by means of the production and application of ideas and content knowledge in real projects (Krajcik & Blumenfeld, 2005: 317).

Álvarez Sánchez (2016: 14) claims that PBL and TBL share some commonalities, but the former stands for a more complex challenge than the latter given that PBL requires more effort and work from the stakeholders. The entire process of carrying out a project is demanding and time-consuming due to the possible integration of various tasks to achieve its completion.

Maths teachers must know how students learn and are able to fully grasp maths concepts. The role of CLIL teachers is not based only on the subject itself but also on the generation of real interactions so that students can put their content knowledge into real practice assuming cognitive challenges. Beswick, Callingham & Muir (2012: 115) mention that mathematics teachers have to recognise maths contexts and applications in order to know how to raise interest and attention to mathematics in unique ways. Therefore, PBL offers maths learners the chance to participate actively in the development of a real project directly linked to maths contents and the possibility of developing language skills in real settings.

3.2.5.4. Flipped classroom

Flipped classroom is an active-based approach that encourages students to learn basic contents outside the classroom. In other words, learning at home based on the materials and resources provided, whereas the class time is used to interact with teachers in order to facilitate and enhance other knowledge acquisition through practice, reflection and understanding in a formative and individualised way, contrary to traditional methods. One asset of this pedagogical model is that allow learners to

assimilate and build their knowledge at their own pace (Ozdamli & Asiksoy, 2016: 99-100; Urfa, 2018: 51).

The swift technological development has made students' learning preferences change (Urfa, 2018: 47). This is one of the reasons why teachers need to use active teaching methodologies. CLIL allows students to join in the learning process and, together with a flipped model, can impact positively on mathematical achievement (Rama Krishnan & Priya, 2016: 62).

3.2.6. Materials

The need for CLIL materials is an aspect that concerns CLIL teachers, given that the availability of didactic materials and appropriate resources for bilingual education stands for a decisive factor to succeed in CLIL programmes. The shortage of CLIL resources interferes in teachers' labour since they have to adjust their workload to be able to design their own materials or modify existing ones such as textbooks (CEFIRE, 2020: 3).

The language of instruction is not only a mere communicative tool; it is part of the learning goals expected in CLIL along with content subject. Using English as the target language supposes a noticeable change in the teaching-learning process, since each teaching/learning strategy and technique applied in the CLIL classroom must be focused on exposure to language and the accomplishment of the curricular content criteria (Bobadilla Pérez & Galán Rodríguez, 2015: 37). Bentley (2010: 50) equally points out that CLIL materials need to be adjusted to the type of contents in accordance with the curriculum, but considering the implications of teaching in a non-native language.

Finding original CLIL materials may be difficult; therefore, some CLIL teachers could tend to confuse them with English Language Teaching (henceforth, ELT) materials. However, in order to avoid these kinds of mistakes, it is crucial to bear in mind that ELT materials are selected according to the needs and requirements of a FL syllabus (Bentley, 2010: 50), not based on the contents of a specific subject as a whole.

Given this lack of specialised materials for CLIL mathematics teaching, content teachers can opt for adapting native-speaking materials and, even though this process may be time-consuming, teachers can gain experience acquiring strong abilities to adapt materials from the internet and from original course books (University of Cambridge, 2011: 7). Bentley (2010: 52) states that some ways to adapt materials are

by including visual aids, diagrams, additional definitions and short explanations as well as by labelling and highlighting key content and vocabulary.

In CLIL mathematics teaching, a method for adapting materials can be translation from L1 textbooks (for instance, from Spanish into English, that is the focus of this dissertation); nevertheless, this process demands mastery of the linguistic competences in the target language and the provision of constant language support and scaffolding. This fact is illustrated by Bobadilla Pérez & Galán Rodríguez (2015: 37, 45, 46), who conducted a study in order to analyse a CLIL section from a mathematics book addressed to CSE and designed by CLIL content teachers in Galicia (Spain). From this case study, these researchers concluded that there is an excessive use of some grammatical patterns and the absence of others that take away some objectivity from the contents. This may be due to the fact that teachers' mother tongue is Spanish, and they are not English experts. However, a fact that stands out is that there is linguistic guidance throughout the unit, which facilitates language understanding in a mathematical context.

CLIL materials have to reflect both content and language learning goals, but specifying all the requirements for guaranteeing a complete comprehension of contents and development of language skills is complex. Notwithstanding, some important aspects to consider before adapting materials from L1 are general vocabulary, grammar, and mathematical terminology (Novotná, Moraová & Hofmannová, 2003: 3-7).

3.2.7. Information and Communication Technologies (ICT)

Technological development has resulted in new educational challenges due to the need to move from traditional educational methods and resources to a new educational stream based on ICT, given that according to Kumar (2008: 556), the application of ICT tools in education shows favourable results in the teaching-learning process by making it more constructive and absorbing.

ICT tools not only comprise computers and the internet, but also television, radio, and other media capable of expanding the access to education, strengthening the skills needed to take part in a digital society, enhancing educational quality, and making education an appealing and active process (Tinio, 2003: 3) through which learners can live real experiences enriching their knowledge and wisdom beyond only theoretical contents.

The integration of technology and CLIL has become a requirement to create a more dynamic and effective learning atmosphere; however, teachers have to demonstrate a set of technological and pedagogical skills to be able to exploit the numerous teaching-learning tools available nowadays (O'Dowd, 2018: 232). For some teachers, the implementation of ICT can signify a real challenge, since many of them are reluctant to leave their comfort zone and open to using innovative methodologies based on the application of ICT in and out of the classroom.

Mathematics teaching either in mainstream education or in CLIL programmes requires to be supported by teaching-learning technologies. Rahman, Ghazali & Ismail, (2003: 1) express that technology reduces time consumption while provides learners with powerful opportunities and ways to enquire into theories and concepts, obtaining higher levels of knowledge than in the past.

O'Dowd (2018: 232) refers to a “virtual exchange” as a positive and fruitful way to engage and motivate pupils in CLIL lessons in which, students can actively partake in a net of collaborative projects under teachers' guidance and support.

3.2.8. Lesson planning

In order to teach mathematics through English as the main language of instruction, it is significant to take into consideration all the implications of planning an effective lesson bearing in mind the duality of CLIL (content and language). Unlike traditional education, in which, lessons focus only and specifically on contents; in CLIL, it is mandatory to have clear notions of the context and the teaching goals on which teachers will be working, in addition to the identification of content knowledge, skills required from pupils, the stages of the lesson and the sequence of the lessons (Bentley, 2010: 30). Needless to say, considering students' language proficiency before and when planning a lesson is paramount.

The good command and planning of the language of instruction aligned to the contents can prevent all the stakeholders from frustration and poor academic performance. Teachers should carefully help students face the challenge of studying through an L2 so that this additional language becomes a positive aspect that increases learning opportunities, and not an obstacle that prevents them from achieving it (Fürstenberg & Kletzenbauer, 2015: 7).

The methodologies and strategies applied in CLIL lessons must be totally opposite to those lecture-based methodologies that put all the attention on teachers, while the students only use receptive skills without a real language production.

Bentley (2010: 30-36) claims that the initial point to plan a lesson is to set learning outcomes based on what students' prior knowledge, abilities and awareness. Furthermore, he details some important facts to bear in mind before planning a lesson:

- Revision of content material: in order to identify possible problems that students could have in terms of language, cognition or task difficulty.
- Use of appropriate materials: all the resources employed in a CLIL lesson must be clear and precise and, if necessary, have to be adapted.
- Objectives, vocabulary and activities on the board: the communication in the target language has to be promoted from the beginning, so that learners can be completely focused on learning.
- Language-rich environment: providing students with as much language in context as possible.
- Task design: motivating assignments, activities and exercises in order to produce subject-specific language.
- Plenary: questions for the end of the class to activate knowledge and interest until the last moment of a lesson.

Similarly, the University of Cambridge (2011: 4-5) establishes some considerations to plan an effective CLIL maths lesson:

- Activating prior knowledge: beginning a lesson by activating schemata helps learners to create connections with the curricular topic to be studied.
- Input and output: a lesson has to be planned in terms of all stakeholders' communication skills in the target language by analysing some aspects such as how to present and deliver the lessons and how students will produce and communicate the subject contents.
- Wait time: this refers to the ideal time to ask students questions and allow them to respond taking into account that the language of instruction is a FL.

- Interactive pair or group work tasks: including pair or group tasks in order to produce subject-specific vocabulary and structures through real interactions and significant learning.
- Cognitive challenge: providing students with enough scaffolding to both everyday and academic language (BICS and CALP) depending on each student's needs.
- Developing thinking skills: teachers should ask questions of varied levels of difficulty in order to foster the development of both lower and higher order thinking skills (LOTS and HOTS) simultaneously.

3.2.9. Teachers' challenges

CLIL stands for a real challenge for practitioners due to its duality, given that focusing on content and language integration is not an easy task at all. Mehisto (2008: 99-100) mentions that one fact that hinders the implementation of CLIL is the lack of knowledge of its theoretical foundations and aims. In this way, CLIL teachers need to be well qualified in this approach and master both content and language pedagogy.

CLIL lesson planning is an integration of knowledge that requires the application of different thinking skills. A CLIL lesson can neither be too easy nor difficult in order to avoid either boredom or frustration. Furthermore, as seen in section 2.2.5, mathematics is in fact a demanding subject to teach and learn and, even more in CSE, where students are more pressured and busy; thus, these facts intensify the challenges to be overcome by CLIL teachers in charge of this subject.

To succeed in CLIL, teachers have to master contents and the target language, a deep knowledge of pedagogy, active methodologies, and the theoretical underpinnings of CLIL. The University of Cambridge (2011: 6) points out that maths teachers need to be able to transmit comprehensible and accurate contents through a good command of English that also helps to check the lexical resource and its correct pronunciation. In addition, teachers have to show an appropriate use of language to ask, give opinions, agree or disagree, clarify information, and foster a real English environment.

3.2.10. Students' engagement

CLIL helps students stay engaged and, certainly, engagement is a key point for effective learning. Recalling that motivation is crucial in order to gain significant

learning, and since the complexity of mathematics and the fact of learning in a different language may cause stress and frustration for learners, teachers must design absorbing lessons and promote an engaging environment that prevents students from losing interest in studying.

Llinares, Morton & Whittaker (2012: 83) state that CLIL involves “students’ cognitive, content and language engagement”. Students’ engagement should be taken into account from three different constructions: cognitive (willingness to learn), affective (confidence, self-esteem and positive emotions), and behavioural (participation development). Motivation and engagement cannot be seen only from an academic perspective, but also emotional (Kong, Wong & Lam, 2003: 5-6).

Students with higher levels of engagement are more consistent and interested in the construction of content knowledge, which also leads to produce more real interactions and establish connections in the target language. Engagement in mathematics learning is pivotal to achieve higher academic achievement.

3.2.11. *Effects of learning mathematics in English as a Foreign Language*

CLIL stands out for allowing students to learn a L2 in an implicit way, unlike ELT in traditional education, in which, English teachers mainly focus on grammar correctness. The focus of CLIL is utterly communicative using a FL as the main vehicle of understanding and communication of contents (Surmont, *et al.*, 2016: 321).

The language of instruction used in mathematics is a matter of concern, since a low language proficiency can affect the understanding of the maths contents, therefore, academic performance (Cuevas², 1984: 134). Learning maths contents while learning English is a double cognitive challenge for pupils, and given the natural complexity of mathematics in L1, mathematics teaching in bilingual education might be detrimental in terms of content knowledge if it is not properly managed.

On the other hand, there has been an extent criticism against CLIL given that the detractors of bilingual programmes claim that students learn neither content subjects nor English (Pavón Vázquez, 2018: 24). However, the success of CLIL may vary according to how CLIL programmes are implemented, whether these meet the necessary criteria or not. Teaching in a L2 is not the same as in mainstream education,

² In the case of Spanish-speaking authors, the second surname is always provided when it is available.

so the shortage of appropriate teacher training might lead students to perform unsuccessfully in CLIL.

Pérez Cañado (2018: 19, 27) states that CLIL appears to influence positively on mathematical achievement. Moreover, she points out that in accordance with her studies to analyse the impact of CLIL in the mother tongue and content learning in three monolingual autonomous communities in Spain (Andalusia, Canary Islands and Extremadura), it is possible to deny that CLIL negatively affects content learning. Even though this conclusion is based on a longitudinal study on natural science in CLIL, in which, CLIL students surpassed the non-CLIL group, in her conclusions, the author does not make a specific mention of the subject.

In a like manner, Husarida & Dollete, (2019: 181-182) believe that English as a medium of instruction (henceforth, EMI), can boost students to enrich both their academic performance in mathematics and language skills; however, a sudden change of the language of instruction can cause unfamiliarity to prior maths knowledge, slow down mathematics learning, and even induce a cultural shock.

In order to prove these statements, Husarida & Dollete (2019: 183, 196) conducted research in an Indonesian private school in order to figure out the level of effectiveness of using English in mathematics and science teaching. From this study, the researchers concluded that EMI was highly effective to enhance learners' mathematical achievement and English language skills, and another key point was the fact that the school had all the facilities to favour the teaching-learning process in a L2.

The perspective of CLIL is wide and, undoubtedly, it is an innovative approach that has rocketed in European bilingual education. The advantages and disadvantages of CLIL have been questioned and its effectiveness continues being controversial, but what is true is that CLIL fosters a real environment to acquire a L2 through content learning. Teachers' expertise is underlying; therefore, they need to be well qualified in all the aspects concerning this approach. Because it is about an integrated curriculum, in which, language and content are equally important, it is not enough to be a content teacher and speak the required L2.

Mathematics is a demanding subject in all languages due to its calculations and procedures, and if the vehicular language in the classroom is English, the challenge is definitely bigger. Notwithstanding, the effectiveness of CLIL depends on all the stakeholders and the proper performance of their roles.

4. LESSON PLAN

4.1. Justification

After having carried out a wide theoretical overview of CLIL mathematics teaching, it is necessary to design a CLIL lesson plan in order to apply the theoretical foundations analysed in this dissertation in a real case such as in an Ecuadorian monolingual public school.

Given that CLIL is still something new and quite unknown in Ecuador, there is little or no research of CLIL in an Ecuadorian context; for that reason, all the literature review has been based on European studies. The main information about CLIL existing in Ecuador is the fact that the Ministry of Education has incorporated CLIL in the Ecuadorian EFL Curriculum as one of the core approaches to attain FL learning outcomes. As a result, this framework is currently being implemented in ELT into primary and secondary schools (Cazco Maldonado & Ortega Auquilla, 2017: 63).

Nevertheless, CLIL goes much beyond only learning EFL; it is the possibility of moving from language learning to language acquisition through content-based instruction, which makes it more interesting and effective. The main objective of this lesson plan is to introduce CLIL into a monolingual context, specifically at San Felipe Neri, a public school in Riobamba, Ecuador, where there is a notorious deficiency of mathematics and English knowledge, as well as a lack of interest in learning them.

By designing an effective and appropriate CLIL lesson plan as a starting point, it is expected to start with a gradual process of improving content knowledge and English language skills. Furthermore, the application of CLIL in a monolingual classroom aims to expand students' motivation and engagement that, as analysed, are fundamental to academic performance.

4.2. Contextualization

San Felipe Neri school is a public educational institution located in an urban area on Saint Amand Montrond avenue and Manuel Quirola street in Riobamba, Ecuador. There are 860 students and 38 teachers, and, compared to other public schools in Riobamba, San Felipe Neri is a small school.

In Ecuador, there are two possible denominations for public educational centres: on the one hand, *escuela* (school) for those centres that offer both primary education and CSE, whereas, on the other hand, the institutions that, in addition to

primary and CSE, also offer baccalaureate education (the counterpart of NCSE) are called *unidades educativas* (educational units). It should be noted that education is compulsory from primary education to baccalaureate in the Ecuadorian educational system. Bearing in mind these facts, San Felipe Neri school's legal name is *Escuela de Educación Básica "San Felipe Neri"*, given that it only offers primary education and CSE.

This school, despite being urban, mostly has students from rural zones, whose parents are mainly farmers or retailers with low incomes. Some students even live in precarious conditions, a fact that has sometimes affected their academic achievement due to the lack of school supplies or having to help their parents work instead of fulfilling their assignments.

The educational model of San Felipe Neri school is guided by the Ecuadorian curriculum, which requires compulsory teaching of language and literature, mathematics, natural science and social science, as core subjects, and complemented by arts, EFL and physical education. Thus, these subjects constitute the curriculum of primary education (*Educación General Básica Elemental y Media*) and CSE (*Educación General Básica Superior*).

In Ecuador, it is possible to find few bilingual programmes in private schools; however, Ecuadorian public education is completely monolingual whereby the goal of planning CLIL maths lessons for San Felipe Neri school is totally experimental, since currently there are no legal regulations for bilingual education. Under these circumstances, the lesson plan will be designed following the guidelines of both the mathematics and EFL curriculum for CSE.

The target group is the tenth grade of CSE (the counterpart of the third grade of CSE in Spain). In this group, there are 25 students (12 boys and 13 girls), who are 14-15 years old. Their academic performance in mathematics and English is worrying due to the fact that they are not attaining the expected learning outcomes in both subjects according to the qualitative and quantitative data from the formative and summative assessment carried out in the first term of the 2019-2020 school year.

4.3. Objectives

4.3.1. General objectives

The general objectives expected to achieve through this lesson plan are set out in the Ecuadorian mathematics curriculum and the EFL curriculum for CSE (*Educación Básica Superior* in Ecuador), respectively.

- To represent and solve graphically (using ICT) and analytically linear equations and simultaneous equations with two variables, as well as to apply them in solving specific situations. REF³. (O.M.4.3.) (my own translation, *Ministerio de Educación Ecuatoriano*, 2016: 889)
- O.EFL 4.5. To introduce the need for independent research as a daily activity by using electronic resources (ICT) in class while practising appropriate competences in the four skills. (*Ministerio de Educación Ecuatoriano*, 2016: 24)

4.3.2. Specific objectives

4.3.2.1. Content objectives

- To create linear equations and simultaneous equations based on the data given in problems.
- To solve maths problems by using linear equations (one variable) and simultaneous equations (two unknowns).
- To understand the main solving methods of simultaneous equations.
- To use ICT tools to graph linear equations.
- To associate maths contents to prior knowledge and experiences.

4.3.2.2. Language objectives

- To understand mathematical vocabulary to maintain effective communication orally and in writing in academic contexts.
- To understand text problems in order to develop reading comprehension.

³ In Ecuador, when a curricular objective has been adapted to a specific context or need, the coding is written at the end preceded by the abbreviation REF. to indicate that the objective has been modified.

- To develop writing and oral production related to maths contents linked to everyday situations and experiences.

4.4. Key competences

4.4.1. Mathematics key competences

These key competences are drafted in the Ecuadorian mathematics curriculum for CSE (my own translation from *Ministerio de Educación Ecuatoriano*, 2016: 882, 884).

CONTENT KEY COMPETENCES
M.4.1.38. Solving linear equations with a variable in R to solve simple problems.
Solving a system of two linear equations with two unknowns algebraically, using the substitution, and Gaussian elimination methods. REF ⁴ . (M.4.1.55.)
M.4.1.56. Solving and posing text problems with sentences that involve linear functions and systems of two linear equations with two unknowns; and interpreting and judging the validity of the solutions obtained within the context of the problem.

Table 4: Content key competences. My own translation from *Currículo de los niveles de educación obligatoria* (Ministerio de Educación Ecuatoriano, 2016: 882, 884)

4.4.2. English key competences

These key competences are drafted in the Ecuadorian EFL curriculum for CSE. (Ministerio de Educación Ecuatoriano, 2016: 25-29)

LANGUAGE KEY COMPETENCES
EFL 4.1.6 Seeking and providing information and assistance, orally or in writing and in online or face-to-face interactions, for personal, social and academic purposes.
EFL 4.1.8 Using suitable vocabulary, expressions, language, and interaction styles for formal and informal social or academic situations in order to communicate specific intentions in online and face-to-face interactions (for example: thanking, making promises, apologizing, asking for permission, chatting with friends, answering in class, greeting an authority figure, etc.).

⁴ In Ecuador, when a key competence has been adapted to a specific context or need, the coding is written at the end preceded by the abbreviation REF. to indicate that the competence has been modified.

EFL 4.2.1 Understanding phrases and expressions related to areas of most immediate priority within the personal and educational domains, provided speech is clearly and slowly articulated (for example: daily life, free time, school activities, etc.).
EFL 4.2.3 Following and understanding short, straightforward audio messages, and/or the main idea/dialogue of a movie or cartoon (or other age-appropriate audio-visual presentations) if delivered slowly and visuals provide contextual support (for example, an announcement of a bus delay, an intercom announcement at school, a dialogue supported by facial expressions/gestures and appropriate intonation, etc.).
EFL 4.2.10 Sustaining a conversational exchange on a familiar, everyday subject when carrying out a collaborative/paired learning activity in which there are specific instructions for a task.
EFL 4.3.5 Using everyday reference material in order to select information appropriate for the purpose of an inquiry and relating ideas from one written source to another.
EFL 4.3.3 Finding specific predictable information in short, simple texts in a range of age- and level-appropriate topics (for example, biographies, news articles, narratives, memoirs and personal accounts, formal letters and emails, etc.).
EFL 4.4.2 Making and using a simple print or digital learning resource to compare and contrast information in order to demonstrate understanding and command of a topic.
EFL 4.4.8 Conveying and organizing information using facts and details in order to illustrate diverse patterns and structures in writing (for example, cause and effect, problem and solution, general-to-specific presentation, etc.).

Table 5: Language key competences. English as a Foreign Language for Subnivel Superior (*Ministerio de Educación Ecuatoriano*, 2016: 25-29)

4.5. Contents

4.5.1. Mathematical contents

The Ecuadorian mathematics curriculum for CSE is divided into three blocks as explained in section 2.2.1; in this specific case, the block of Algebra and functions is the main focus of this lesson plan as detailed below.

ALGEBRA AND FUNCTIONS	
Topic	Subtopics
Equations	Linear equations • Graphic solution • Analytical solution
	Simultaneous equations • The graphical method • The substitution method • The elimination method

Table 6: Mathematical contents for CLIL lesson plan. Following *Currículo de los niveles de educación obligatoria* (Ministerio de Educación Ecuatoriano, 2016: 224-227)

4.5.2. Language contents

The language contents to be reviewed refer to the communicative skills needed to perform successfully in the CLIL classroom both in social interactions and academic contexts. In this way, the main language focus for the students of the tenth grade of CSE is described in the table.

LANGUAGE CONTENTS
• Vocabulary related to algebra: elimination method, equation, equals sign, formula, graphical method, like terms, simultaneous equations, substitution method, term, variable, etc. • Reading comprehension of text problems. • Oral production to describe procedures. • Writing skills to describe maths relationships and procedures.

Table 7: Language contents for CLIL lesson plan. Following *Currículo de los niveles de educación obligatoria* (Ministerio de Educación Ecuatoriano, 2016: 224-227)

4.6. Methodology

CLIL requires the application of student-centred methodologies in order to put learners as the main builders of their knowledge under teachers' guidance. Considering all the key points of the effective methodologies reviewed in section 3.2.5 to teach mathematics through English, and bearing in mind the background of the students of San Felipe Nery school, the proper methodologies to introduce CLIL in a monolingual environment are Flipped Classroom, TBL, and Cooperative Learning.

4.7. Materials

In order to design the necessary materials, all the considerations detailed in section 3.2.6 will be taken into account as well as the importance of using ICT in the classroom (section 3.2.7). These didactic materials needed to carry out CLIL maths lessons must provide the group of students in question with all the resources to perform successfully in mathematics learning through CLIL as well as developing language skills.

Given that the students of the tenth grade of CSE of San Felipe Neri school have never been exposed to any kind of content-based instruction, the CLIL materials have to be focused on their academic and personal context respecting the 4Cs framework defined by Do Coyle. These materials will be specified in each session.

4.8. Attention to diversity

Catering to diversity involves guaranteeing quality education, and the Ecuadorian Ministry of Education promotes inclusive education in the Ecuadorian curriculum for all educational levels and sublevels. Teachers are responsible for adapting the learning environment, resources, and strategies to students' needs and background in order to provide significant learning despite any educational needs associated or not with disability.

In the context of the target group for this lesson plan, there is no educational need associated with disability; however, there are some behavioural, emotional, and social difficulties that must be handled carefully. Six students need more attention and help to focus on classes; therefore, individual activities will require more guidance and supervision for those students. In addition, working groups will be arranged according to each student's abilities to manage different student profiles in each so that cognitively stronger students can support their peers.

Given the low socioeconomic status of some students, most of them do not have devices such as computers or access to the Internet that can help them complement their learning process. In this way, it will be necessary to exploit the computer lab available at school to enhance students' digital competences whereby they can benefit from a more dynamic and interactive teaching-learning process.

Overall, all the students need strategies that help them to increase self-esteem and confidence in subjects considered as difficult (that is the case of both mathematics and EFL). For that reason, it is important to provide them with maths instruction at the

grassroots whereby the students can evolve from their prior knowledge and advance in learning by means of a L2 (authentic use of language).

4.9. Step-by-step account of the sessions

4.9.1. Session 1: Introduction to equations

Topic: Introduction to equations		Time: 40 minutes ⁵
Aim: To review basic theory about equations.		
TEACHING OBJECTIVES		
Content	Cognition	Culture
Addition Equations Elements of equations Subtraction Multiplication Division	Identifying missing values to solve mathematical identities. Comparing and contrasting operations in equations.	Self-confidence to express their own ideas in a FL
Communication		
Language of learning	Language for learning	Language through learning
Key vocabulary: Add, addition, divide, division, equation, equals sign, equation, multiplication, multiply, subtract, subtraction, term. Maths vocabulary.	Identifying: What is this? This is ... Justifying: Why is this an equation? This is an equation because...	Classroom language Extending presentation skills

⁵ In Ecuador, an academic hour lasts 40 minutes.

LEARNING OUTCOMES		
Content		Language
Learners will be able to: - Identify equations. - Write simple equations from text problems.		Learners will be able to: - Say the definition of “equation”. - Listen to, read and understand the basic construction of equations.
PROCEDURE		
Time	Activities	Materials/ resources
5’	Brainstorming: What does the equals sign mean? Video: Understanding the equal sign + addition and subtraction equations.	Video: https://tinyurl.com/ya234cgh (3’30’')
6’	Examples: students write 5 examples of equations similar to those provided in the video. Pair work: a student dictates some equations to his/her partner, as he/she copies what he/she hears. After finishing, they interchange positions, and check their hits and misses.	Pencil/pen Notebook
14’	Completing equations: students look at the equations and determine what the missing value is. Besides, they write the name of the main operation of each equation. Interpreting textual equations: students write the texts and translate them into equations.	WS ⁶ 1
15’	Crossword: students read the clues to find the correct answer. Maths glossary: students write down the words and the definitions they have learnt during the class.	WS 2
ASSESSMENT		
- Content is assessed bearing in mind the students' ability to write and solve equations. - Language is assessed in terms of listening comprehension, oral production to express ideas, and reading comprehension to understand statements.		

Table 8: Session 1 (my own creation).

⁶ WS is, henceforth, the abbreviation used in the lesson plan for the word “worksheet”.

Worksheet 1: Equations

Equations

1. What is the value of x in each equation? Write the main operation you see.

$3 + x = 7$

$6x = 12$

$4 - x = 10$

$2x = -10$

$15 + x = -6$

$25 - x = 8$

$14 + 3x = 20$

$5x - 5 = 10$

2. Write the following texts as equations.

- Five added to seven is equal to twelve.

- Three multiplied by two, then added to ten is equal to sixteen.

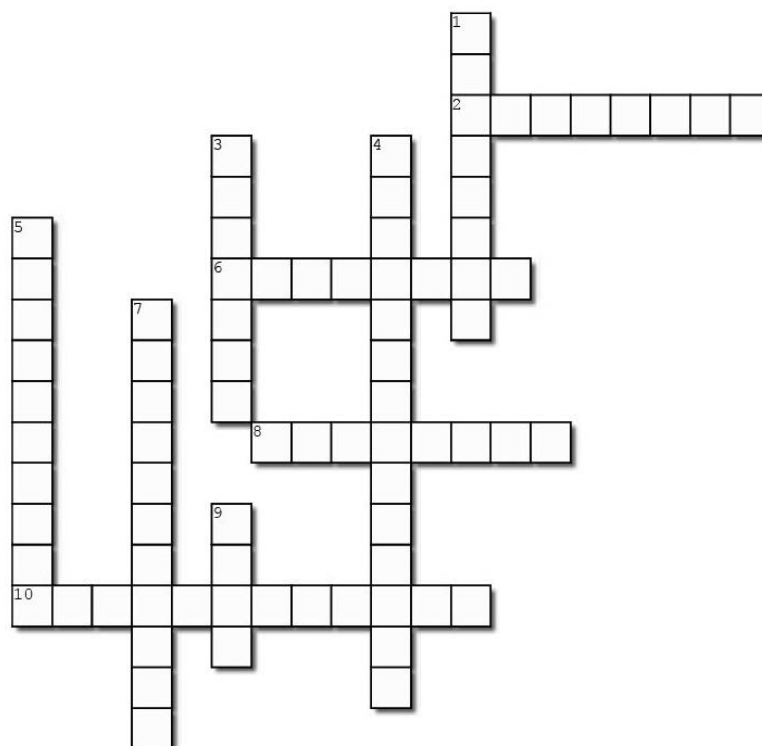
- Eighteen divided by nine is equal to two.

- Fifteen subtracted from twenty-five, then added to five is equal to fifteen.

Figure 3. Worksheet 1: Equations (my own creation)

Worksheet 2: Crossword

Math vocabulary



Across

- 2. The process of dividing a quantity by another under specific rules to obtain a quotient.
- 6. A statement that the values of two mathematical expressions are equal.
- 8. A letter used to represent a number.
- 10. the process of taking a quantity away from another under specific rules to obtain the difference.

Down

- 1. The process or skill of calculating the total of two or more numbers or amounts.
- 3. The part of mathematics in which letters are used to represent numbers.
- 4. The process of combining quantities under specific rules to obtain their product.
- 5. They have the same letter or symbol.
- 7. A mathematical symbol used to indicate equality.
- 9. Each part of a line of algebra.

Figure 4. Worksheet 2: Maths vocabulary (my own creation through The Teacher's Corner).

4.9.2. Session 2: Linear equations I

Topic: Linear equations I		Time: 40 minutes
Aim: To use equations to represent simple problems.		
TEACHING OBJECTIVES		
Content	Cognition	Culture
Use of variables. Resolution of equations. Simple problems based on equations.	Understanding how to solve linear equations. Interpreting simple problems to translate them into equations.	Linking mathematics and everyday language; recognising some similar words in English and Spanish.
Communication		
Language of learning	Language for learning	Language through learning
Key vocabulary: Addition, division, equals sign, inverse operations, linear equation, multiplication, subtraction. Language to translate statements into algebraic expressions. Language to describe maths procedures.	Reading skills to identify the variable in written problems. Explaining maths problems orally.	Dictionary skills. Extending presentation skills. Classroom interaction.
LEARNING OUTCOMES		
Content	Language	
Learners will be able to: Solve linear equations.	Learners will be able to: - Explain written and orally how to solve equations. - Translate simple short problems into equations.	

PROCEDURE		
Time	Activities	Materials/ resources
5'	Warm-up: Opposing friends. Finding inverse operations.	Set of cards (Sample 1)
25'	Video: Algebra video for kids: Solve equations with variables. Pair-work: students write the process to solve a linear equation. Oral presentations: Three couples of students are chosen randomly to describe the process of solving equations. Each couple has to write some examples on the whiteboard and solve and explain them orally.	Video: https://tinyurl.com/wcg5oj2 (4'02") English-Spanish dictionary
10'	Solving equations: Students read the statements to translate them into equations and solve the equations analytically. Students have to check their answers by graphing the equations in Geogebra.	WS 3 Geogebra: https://tinyurl.com/yamm3ccq
ASSESSMENT		
- Content is assessed based on the correct resolution of linear equations. - Language is assessed considering their listening comprehension, written and oral production to describe maths procedures, and reading comprehension of maths statements.		

Table 9: Session 2 (my own creation).

Sample 1: Opposing friends

This is a warm-up to remind students which maths operations are inverse, this being basic knowledge to solve linear equations. First of all, students are given a sheet with a set of six cards (cf. figure 5), which have the names of the maths operations. After that, the teacher indicates that there will be six rounds, in which, each student has to select a card and seek a partner who has the inverse operation. For instance, in the first round, a student chooses a card that reads “multiplication”; then, he/she has to look for a classmate that has a card that says “division”. All students who have not been able to find the reverse operation should sing a song or dance a choreography

in the next class as a punishment. Students cannot choose the same maths operation twice.



Figure 5. Set of cards (my own creation).

Worksheet 3: Linear equations

Linear equations

Read the statements, translate them into equations and solve them.

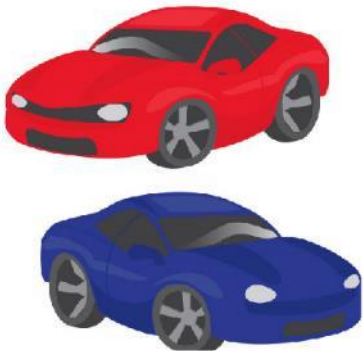
If I have 35 books, but my brother and sister borrow 22, how many books do I have left?



Source: <https://tinyurl.com/y85hjcnc>

A large, empty rectangular box with a pink border, intended for the student to write the equation for the book problem.

While Eduardo was driving his car on the highway, he saw twice as many red as green cars. If he saw 15 cars in total, how many red cars did he see?



Source: <https://tinyurl.com/y9d8z6se>

A large, empty rectangular box with a red border, intended for the student to write the equation for the car problem.

Figure 6. Worksheet 3 (my own creation).

4.9.3. Session 3: Linear equations II

Topic: Linear equations II		Time: 40 minutes
Aim: To understand and solve text problems through linear equations.		
TEACHING OBJECTIVES		
Content	Cognition	Culture
Use of variables. Resolution of equations. Problems based on equations.	Understanding how to solve linear equations. Interpreting problems to solve them through linear equations.	A positive attitude and behaviour towards mathematical language in English.
Communication		
Language of learning	Language for learning	Language through learning
Key vocabulary: Added to, double, equations, multiplied by, take away, triple, twice. Imperative mood to give instructions. Simple present tense to describe procedures. Passive voice for impersonal actions.	Writing mathematical problems. Asking and answering in pair work.	Classroom language. Recycling discussion skills.
LEARNING OUTCOMES		
Content	Language	
Learners will be able to: Solve problems through linear equations.	Learners will be able to: Read/listen to and understand text problems to write equations.	

PROCEDURE		
Time	Activities	Materials/ resources
5'	Quiz: Algebraic language. Feedback to the assignment.	Computer lab Kahoot: https://kahoot.it/ Pin: 09132830 (Sample 2)
10'	Independent work: Students formulate a problem to be solved through the equation $3x - 6 = 39$. After that, three students have to solve and expose it.	Pencil/pen Notebook
20'	Cooperative work: I tell you, you solve. In pairs, students receive a handout with two problems. Student A reads the problem 1, while student B listens to his/her partner, understands the problem and writes the equation to solve it. After that, both students interchange positions to continue with problem 2. After finishing, each student answers the self-assessment questions that are under his/her solved problem.	WS 4
5'	Song: Solving Equations Students listen to and sing a song by reading the lyrics in the video that summarises how to solve equations.	Song: https://tinyurl.com/yd6hqylc (1'50")
ASSESSMENT		
- Content is assessed based on the correct resolution of linear equations. - Language is assessed taking into account written and oral production to describe maths procedures as well as listening and reading comprehension to solve text problems.		

Table 10: Session 3 (my own creation).

Sample 2: Kahoot quiz

Students type a pin to enter the game “Algebraic language”, and self-assess their interpretation of written expressions into algebraic language. The expression is presented, students have to select the correct answer, and they get immediate feedback since they can see the correct answer.

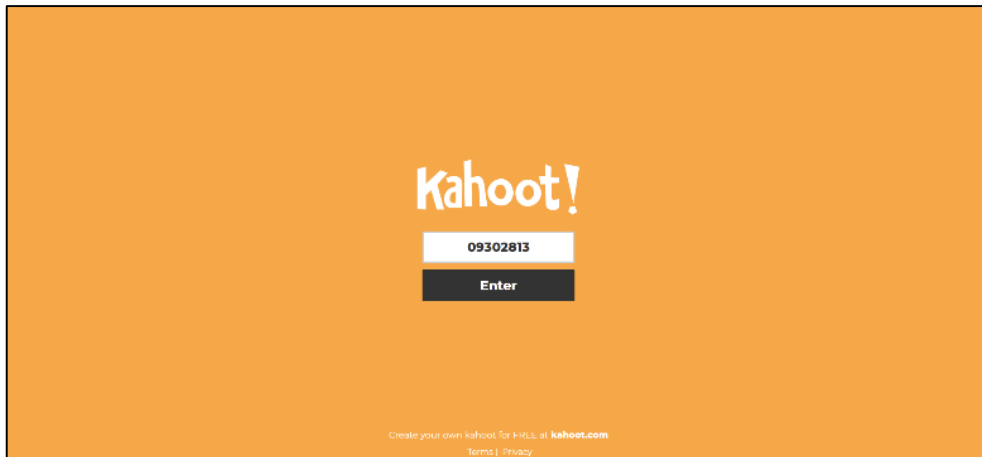


Figure 7. Kahoot's screen (my own creation through Kahoot).

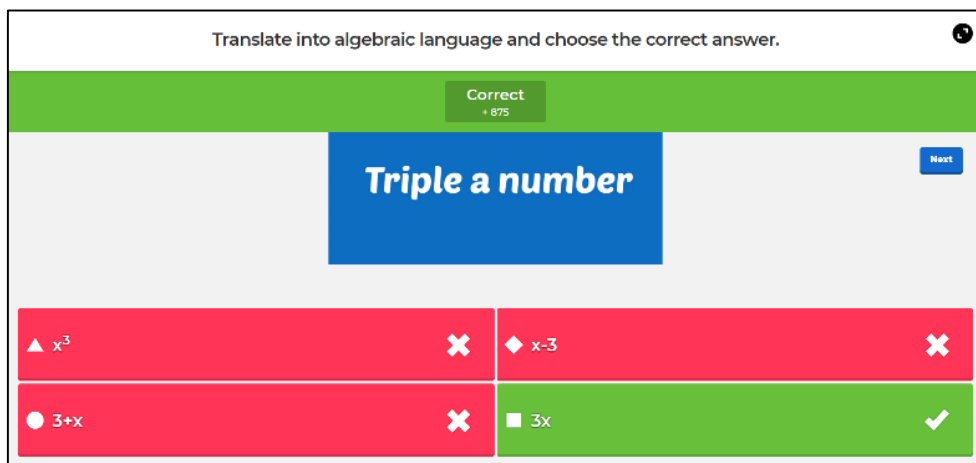


Figure 8. Sample 1 out of 8 questions (my own creation through Kahoot).

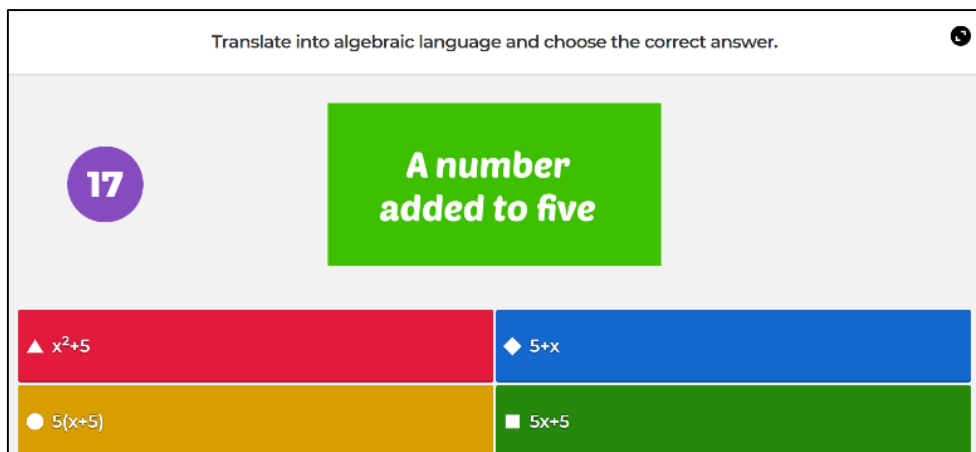


Figure 9. Sample 2 out of 8 questions (my own creation through Kahoot).

Worksheet 4: I tell you, you solve

I tell you, you solve

1. If at the age of Susana, its third part is added, the age of Alberto is obtained. How old is Susana if Alberto is 36 years old?

After solving this problem, I say that ...

SELF-ASSESSMENT	YES	NO	I'M NOT SURE
I can solve linear equations.			
I can solve problems through equations.			
I understand the importance of equations in daily life.			

2. The difference between two numbers is 30. When we decrease the largest of them by 14, we get triple the least. Which are the numbers?

After solving this problem, I say that ...

SELF-ASSESSMENT	YES	NO	I'M NOT SURE
I can solve linear equations.			
I can solve problems through equations.			
I understand the importance of equations in daily life.			

Figure 10. Worksheet 4 (my own creation).

4.9.4. Session 4: Introduction to simultaneous equations

Topic: Introduction to simultaneous equations		Time: 40 minutes
Aim: To understand and identify simultaneous equations.		
TEACHING OBJECTIVES		
Content	Cognition	Culture
Definition and characteristics of simultaneous equations. Examples	Identifying and analysing simultaneous equations.	Simultaneous equations as mathematical representations of everyday situations.
Communication		
Language of learning	Language for learning	Language through learning
Key vocabulary: Linear equations. simultaneous equations, systems of equations, unknown. Language of defining. Language of describing.	Asking and answering questions. Agreement/disagreement	Classroom language. Recycling discussion skills.
LEARNING OUTCOMES		
Content	Language	
Learners will be able to: Identify simultaneous equations, unknowns and examples.	Learners will be able to: Give oral and written definitions.	
PROCEDURE		
Time	Activities	Materials/ resources
5'	Memory game: Finding simultaneous equations.	Educaplay: https://tinyurl.com/y9ljb9r (Sample 3)

15'	Discussion: Based on what was reviewed in the game, what characteristics do simultaneous equations have? Video: Simultaneous equations introduction.	Video: https://tinyurl.com/ybbd34nv (4'26")
10'	Definition of simultaneous equations: With their own words, students write what simultaneous equations are and write an example.	English-Spanish dictionary Pencil/pen notebook
5'	Summary: By collecting the main ideas reviewed during the class, the students and the teacher summarise them on the whiteboard by creating a mind map.	Whiteboard Markers
ASSESSMENT		
- Content is assessed considering the level of understanding of simultaneous equations. - Language is assessed in terms of oral production to discuss a topic, listening comprehension, and written production to define what simultaneous equations are.		

Table 11: Session 4 (my own creation).

Sample 3: Memory game in Educaplay

This memory game has been created to drive an initial understanding of simultaneous equations. The students have to match equations to make systems of equations. In this way, they can analyse and build their knowledge of the subject in a fun way.

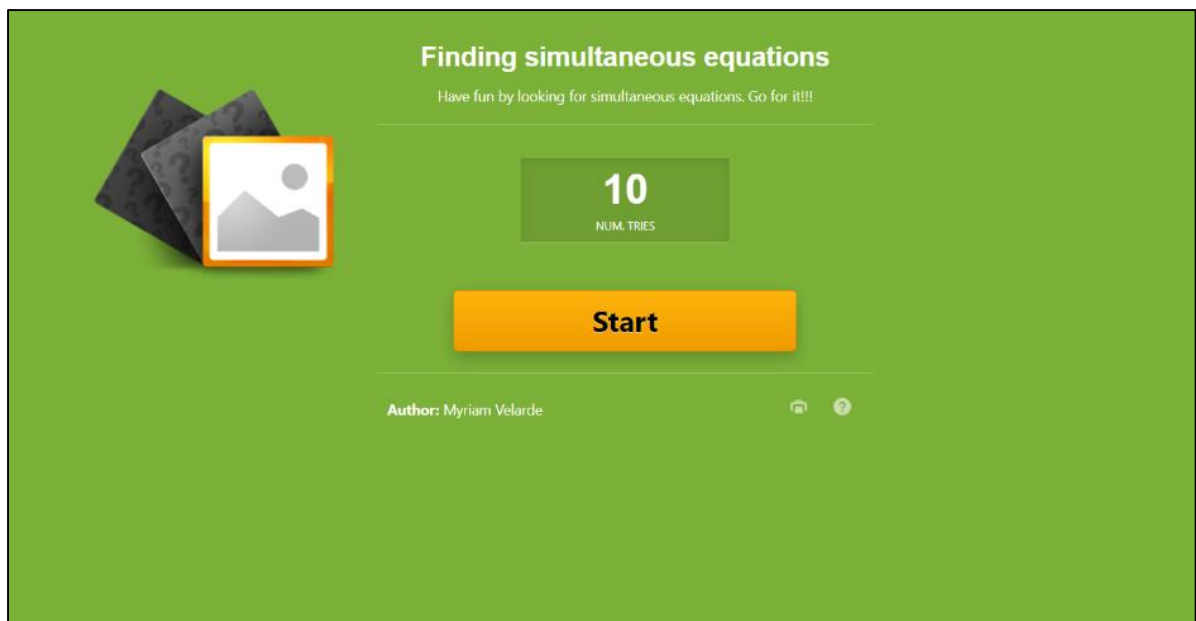


Figure 11: Memory game home screen (my own creation in Educaplay).

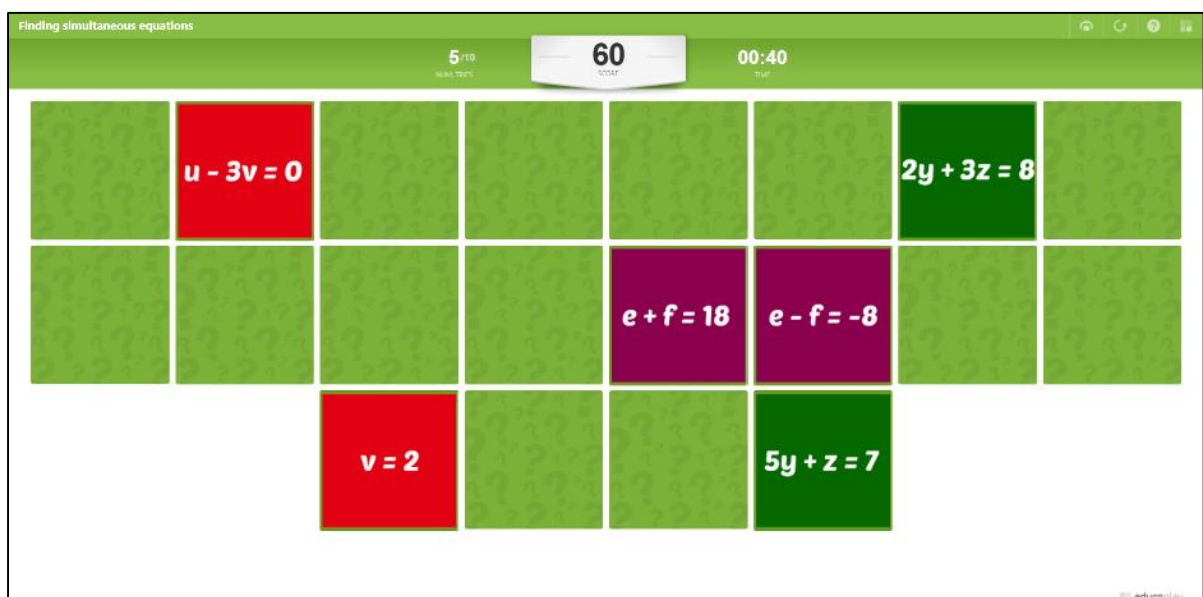


Figure 12: Memory game (my own creation through Educaplay).

4.9.5. Session 5: Simultaneous equations: graphical method

Topic: Simultaneous equations: graphical method		Time: 40 minutes
Aim: To learn how to solve simultaneous equations graphically.		
TEACHING OBJECTIVES		
Content	Cognition	Culture
Solving simultaneous equations by graphing. Use of interactive maths applications. Problem solving.	Understanding the process to solve simultaneous equations by graphing.	The use of technological tools for learning and fun.
Communication		
Language of learning	Language for learning	Language through learning
Key vocabulary: Coefficient, coordinate system, Geogebra, plot, simultaneous equations, unknown, x-axis, y-axis.	Writing procedures. Asking and answering questions. Explaining calculations.	Classroom language. Extending speaking/listening skills.
LEARNING OUTCOMES		
Content	Language	
Learners will be able to: Graph and solve simultaneous equations through ICT.	Learners will be able to: - Describe how to solve simultaneous equations. - Interpret problems based on simultaneous equations.	
PROCEDURE		
Time	Activities	Materials/ resources
9'	Word search: Students review basic maths concepts.	WS 6
6'	Video: How to solve simultaneous equations graphically.	Video: https://tinyurl.com/y8hfgy34 (5'12")

5'	In pairs: Students discuss the process to solve simultaneous equations graphically. One student explains the main points while his/her partner writes them down in his/her notebook.	Pencil/pen notebook
20'	Independent work: Graphing and exploring. Students have to use Geogebra to graphically solve six systems of equations. Each system has to be solved correctly to be able to move forward in the game.	Gamilab: https://tinyurl.com/y9k378b6 (Sample 4) Geogebra: https://tinyurl.com/yamm3ccq
ASSESSMENT		
- Content is assessed in terms of the understanding of graphing systems of equations to solve them. - Language is assessed taking into account listening and reading comprehension to understand definitions and solve problems, oral production to discuss a topic, and written production to describe math procedures.		

Table 12: Session 5 (my own creation).

Sample 4: Graphing and exploring

Through the use of ICT, students solve some simultaneous equations, which are presented by means of a game created in Gamilab. This game is based on exploring different nodes that show the tasks to be solved sequentially, since if a task is not correctly solved, the student cannot advance towards the next exercises.

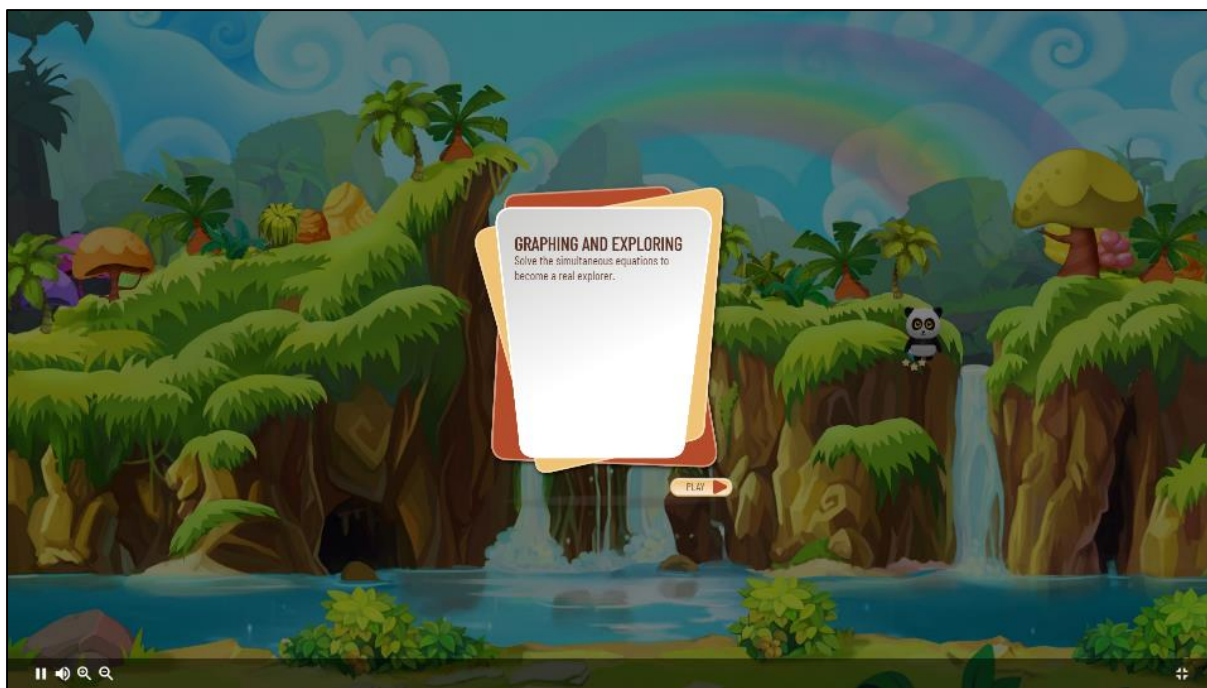


Figure 13: Start screen in Gamilab (my own creation through Gamilab).

The aim is to explore the different nodes that present a problem based on simultaneous equations, which have to be solved graphically by using Geogebra, an interactive maths application.

Calculate a number knowing that the sum of its two figures is 10; and that, if we reverse the order of these figures, the number obtained is 36 units greater than the initial one.

The number is 28

The number is 40.

The number is 37.

FINISH

Figure 14: Sample 3 out of 6 questions (my own creation through Gamilab).

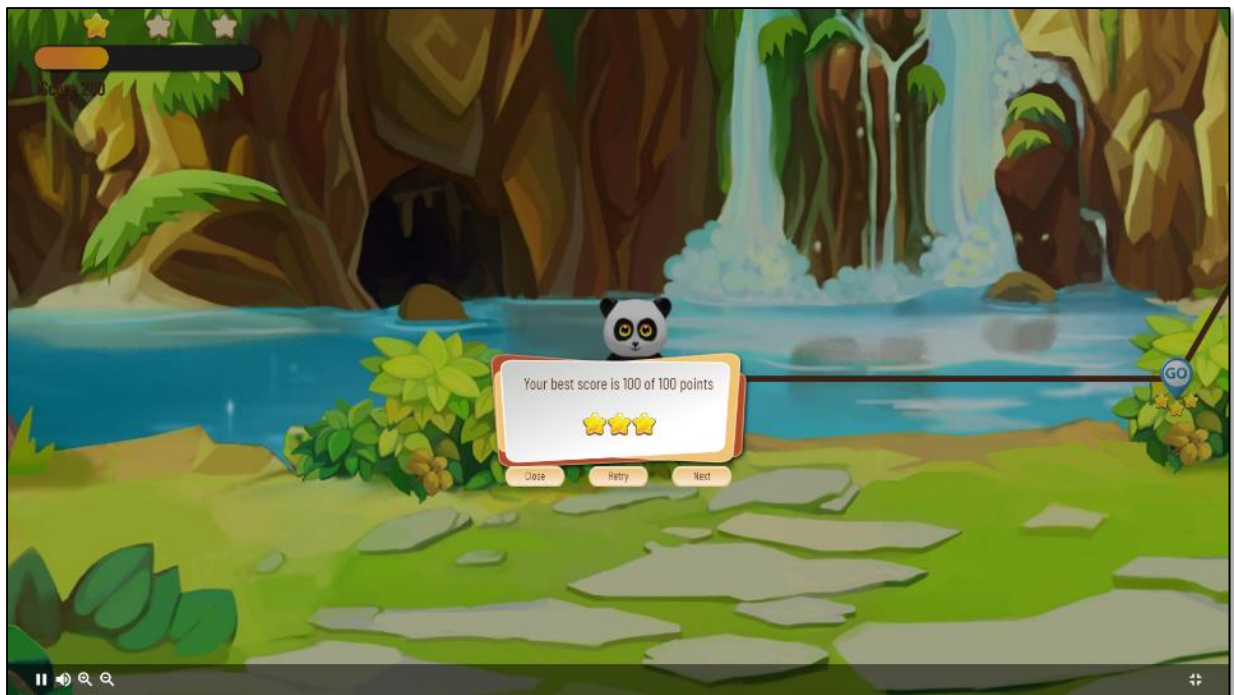


Figure 15: Task score (my own creation through Gamilab).

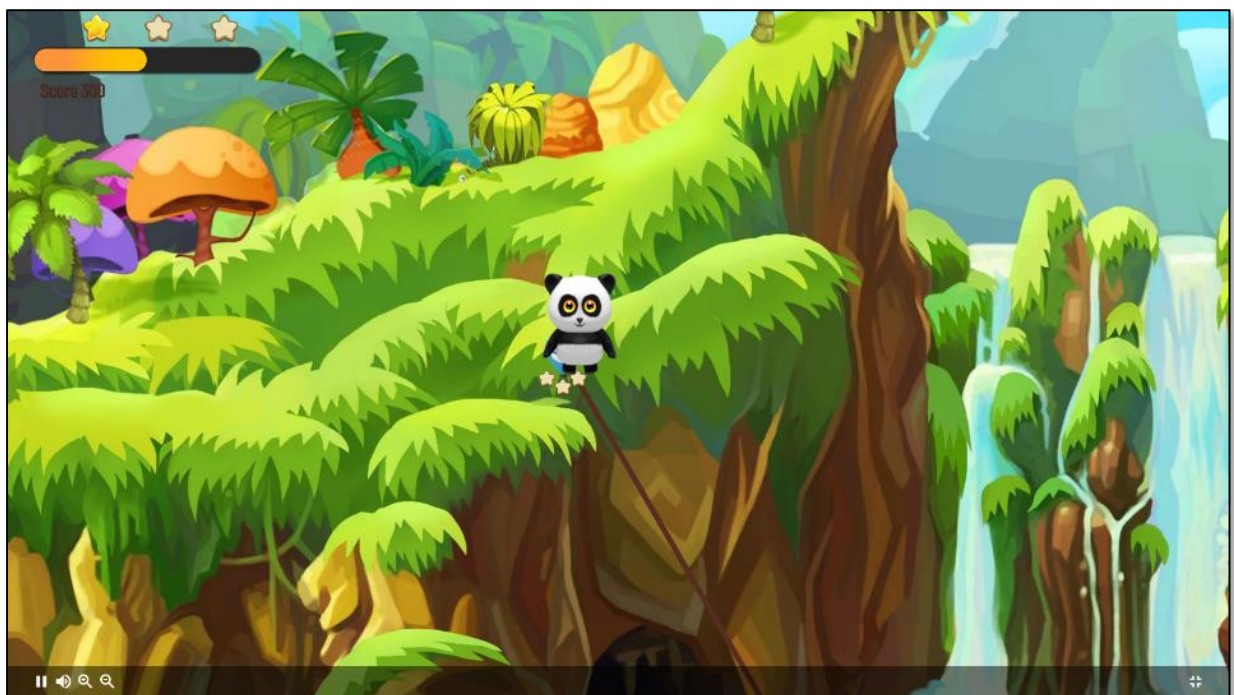


Figure 16: Exploration progress (my own creation through Gamilab).

Worksheet 5: Searching maths concepts

Searching maths concepts



1. Any exact point on the graph is called _____
2. _____ is a fixed line against which the positions of points are measured, especially points on a graph.
3. _____ is a software used for learning and teaching mathematics.
4. _____ have two equations with two _____ values
5. _____ is a number that is placed before another quantity and that multiplies it.
6. _____ is a system that uses one or more numbers, or coordinates, to uniquely determine the position of the points or other geometric elements.
7. _____ is a diagram consisting of a line or lines, showing how two or more sets of numbers are related to each other

Figure 17: Worksheet 5 (my own creation through Puzzlemaker).

4.9.6. Session 6: Simultaneous equations: elimination method

Topic: Simultaneous equations: elimination method		Time: 40 minutes
Aim: To use the elimination method to solve simultaneous equations.		
TEACHING OBJECTIVES		
Content	Cognition	Culture
Solving simultaneous equations by means of the elimination method. Problem solving.	Understanding the process of the elimination method to solve simultaneous equations. Identifying the advantages and disadvantages between the elimination and graphical methods.	The importance of games in learning.
Communication		
Language of learning	Language for learning	Language through learning
Key vocabulary: Double, half, simultaneous equations, variable, unknown. Advanced language to translate text problems into algebraic expressions. Passive voice to describe impersonate actions.	Asking/answering questions. Language to build agreement/disagreement. Language to give opinions.	Using feedback. Classroom language. Development of communication skills.
LEARNING OUTCOMES		
Content	Language	
Learners will be able to: - Use the elimination method to solve simultaneous equations. - Solve problems through simultaneous equations.	Learners will be able to: - Answer questions on the topic reviewed. - Interact with their partners to solve problems and tasks.	

PROCEDURE		
Time	Activities	Materials/ resources
10'	Warm-up: Simultaneous equations with the body Students are divided into two groups to play a game based on trying to write equations with their bodies.	(Sample 5)
10'	Video: Simultaneous equations by elimination. Students watch the video attentively, and then, they answer the following questions in their notebooks. • What is the method used in the video? • What are the disadvantages of using the graphical method compared to this new method? • Is this method useful for all kinds of equations? • What is the purpose of the method in question?	Video: https://tinyurl.com/yb77sfry (6'22'')
20'	Game: Simultaneous equations race In pairs, students participate in an virtual bike race, along which, they have to solve problems by using the elimination method to win the race.	Gamilab: https://tinyurl.com/yazftufb (Sample 6)
ASSESSMENT		
• Content is assessed based on the application of the elimination method to solve problems. • Language is assessed considering listening and reading comprehension to solve problems, written production to answer questions, and oral production to interact among peers.		

Table 13: Session 6 (my own creation).

Sample 5: Forming simultaneous equations

Firstly, the total number of students is divided into two teams, in which, a leader has to be chosen. Secondly, the teacher writes a system of equations on the whiteboard, then each team has to form the system by means of the ability of improvisation of each student to make a letter, a number or a maths sign with his/her body (cf. figure 18); each leader has to check the correct completion of the task. At the end, the team that finishes first wins and decides a punishment for the other team.



Figure 18: Writing with the body. (Source: <https://tinyurl.com/yamr7shh>)

Sample 6: Simultaneous equations race

This is a bike race created in Gamilab, through which, the students work in pairs to win the race. The interface of the game is in 3D, this being an aspect that motivates them to participate in this activity. Along the race, the students find problems based on simultaneous equations that have to be solved in order to get more speed and be able to finish the race faster.

Each problem is a multiple-choice question, and immediately, the students can know if they answered correctly or not, which promotes self-assessment. At the end, the game shows a high score table to see the position and the time employed by all the competitors. In the game settings, the teacher can see the number of hits and misses.

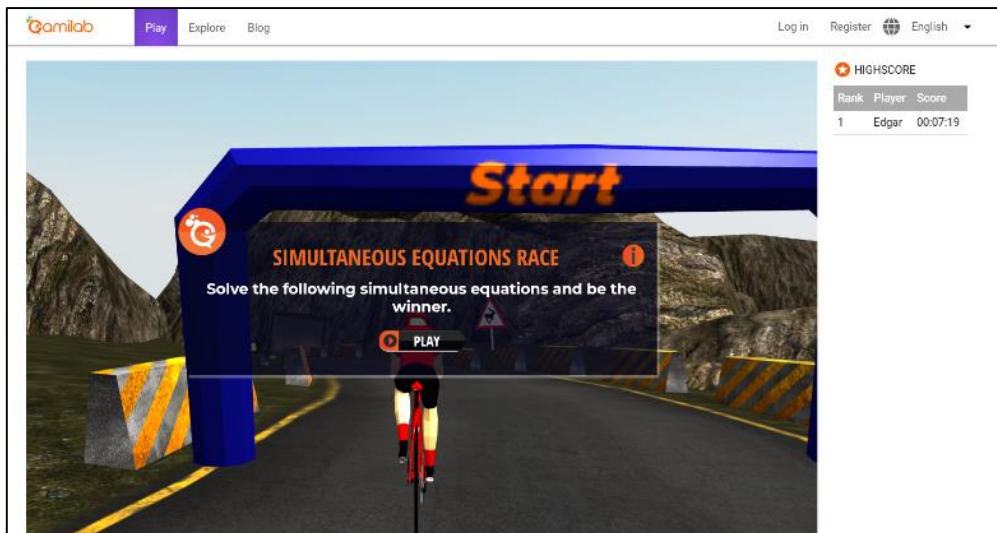


Figure 19: Star screen of the game (my own creation through Gamilab).



Figure 20: Bike race (my own creation through Gamilab).

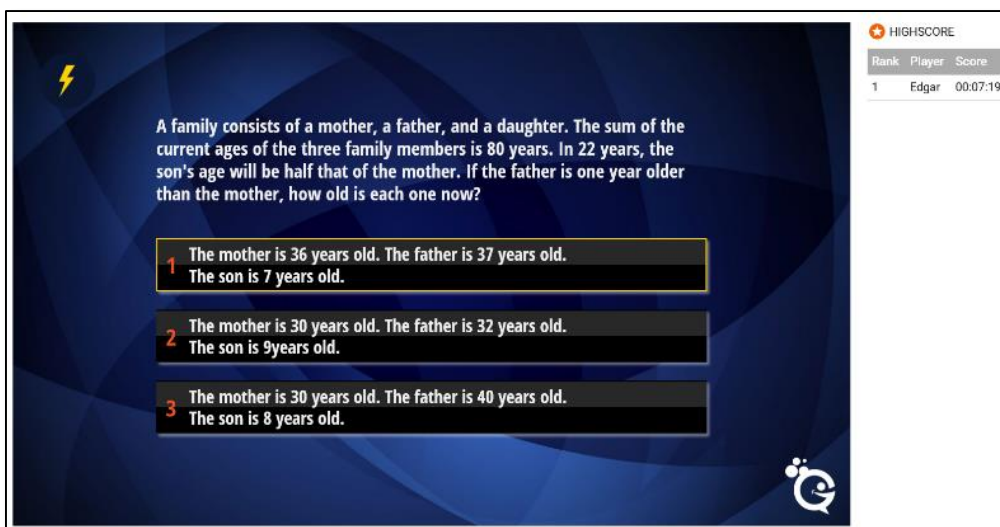


Figure 21: Sample 1 out of 8 questions (my own creation through Gamilab).

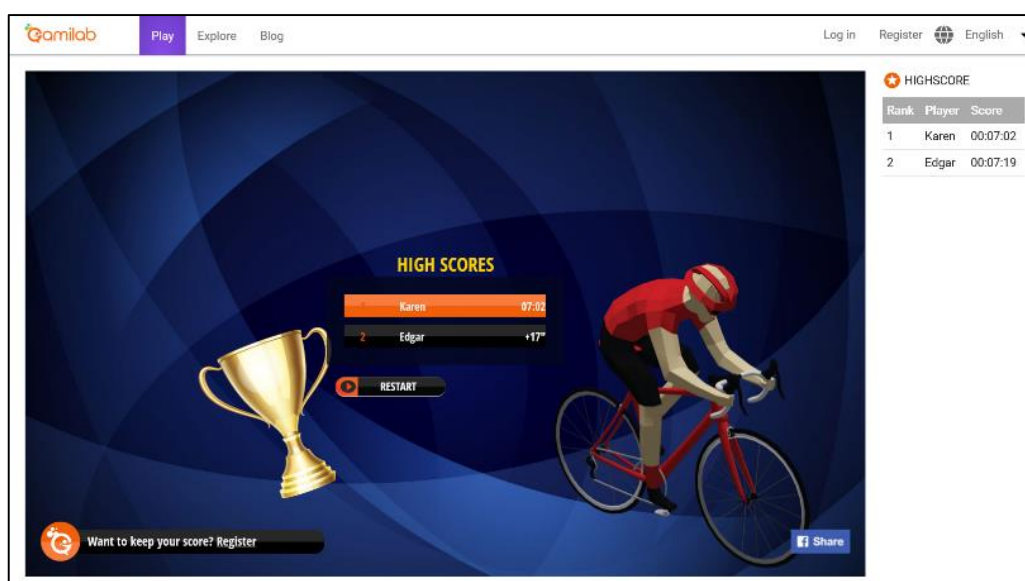


Figure 22: High score table (my own creation through Gamilab).

4.9.7. Session 7: Simultaneous equations: substitution method

Topic: Simultaneous equations: substitution method		Time: 40 minutes
Aim: To use the substitution method to solve simultaneous equations.		
TEACHING OBJECTIVES		
Content	Cognition	Culture
Solving simultaneous equations by means of the substitution method. Problem solving.	Understanding the incidence of equations in daily life. Applying the process of the substitution method to solve simultaneous equations.	The interaction in a foreign language in common situations.
Communication		
Language of learning	Language for learning	Language through learning
Advanced language to make describe procedures. Linking words.	Asking/answering questions. Language to give opinions. Language to propose ideas.	Classroom language. Development of communication skills.

LEARNING OUTCOMES		
Content		Language
Learners will be able to: • Use the substitution method to solve simultaneous equations. • Solve problems through simultaneous equations.		Learners will be able to: • Express the importance of equations. • Summarise math procedures accurately.
PROCEDURE		
Time	Activities	Materials/ resources
10'	Reading: Why equations are important in our life.	WS 7
10'	Video: Systems of equations: Substitution method Writing: Based on the video, students briefly describe the substitution method in her notebooks. At the end of the activity, two students are selected to explain the topic to the rest of the class.	Video: https://tinyurl.com/y926t8mk (5'20'')
20'	Work-group: Helping at the restaurant Students are organised into groups of 5 to solve a task and find out how much each dish costs by solving simultaneous equations.	Cards: Task The waiter only knows that... Menu Bill (Sample 7)
ASSESSMENT		
• Content is assessed based on the application of the substitution method to solve problems. • Language is assessed taking into account listening and reading comprehension of problems and maths procedures, written production to describe methods, and oral production to interact among peers.		

Table 14: Session 7 (my own creation).

Worksheet 7: Why equations are important in our life

Why equations are important in our life

Why do we have to learn math equations? Why equations are important in our life? So many other questions arise in students mind. If you think equations are pure math or physics, then you are wrong. Equations are actually used by us in daily life. As we know most of the mathematics is depends on equation and mathematics is one of the disciplines which we utilize in everyday life. Thus, we use equations very frequently in daily life.

An equation is the mathematical representation of those two things which are equal, one on each side of an 'equals' sign. Equations are useful to solve our daily life problem. Most of the times we take pre algebra help to resolve real life problems. Pre-algebra concepts are the very basics of math.

Today's computer chips used in all the machines we use in daily routine like washers, dryers, cars, backs, etc. All chips, which we use in these machines based on mathematical equations and algorithms.

We use the internet to look up the information. We just type the words and get many websites related to that word. Do you ever think the process behind this internet search? This is all the parts of mathematics equations, which allow us to get information from all over the world within seconds. For example, let you search for free online tutoring within seconds, you get N number results. This is a part of the math equation. The equation for this is, "Resulting sites with (free online tutoring) = input words (free online tutoring)".

Mathematical equations are also used in traffic control, aircraft, space program and medicine and so on. So we should always remember that any math equation result has the potential to change the world. That is the reason all mathematical equations are important in our life.

Marsh, J. (2014). *Why Equations Are Important in Our Life*. Retrieved from <https://n9.cl/equationsareimportant>

After reading, answer the following questions.

1. Do you agree that equations are important for our lives? Why?

2. Which all the chips we use are based on?

3. What is the relationship between the Internet and equations?

4. Based on the reading, what are the equations used for?

Figure 23: Worksheet 7 (based on Marsh, 2014).

Sample 7: Helping at the restaurant

The students are divided into 5 groups to work on a task, which is presented as a situation in a restaurant where the menu does not show the prices of each dish, and the waiter does not know the prices either, so each group of students assumes the role of customers who have to help the waiter determine the food prices and the total bill that they must pay by using simultaneous equations formulated from the waiter's information.

Each group receives a set of cards that is made up of the task (cf. figure 24), the information that the waiter knows (cf. figure 25), the menu (cf. figure 26) and the bill (cf. figure 27). In these last two cards, the students must fill in the blank spaces with the prices obtained from the application of simultaneous equations.

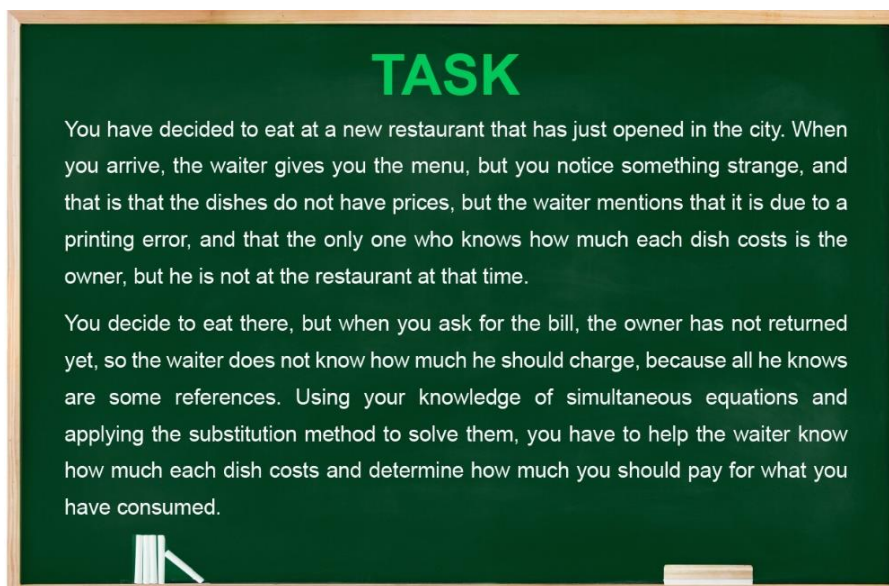


Figure 24: Description of the task (my own creation).

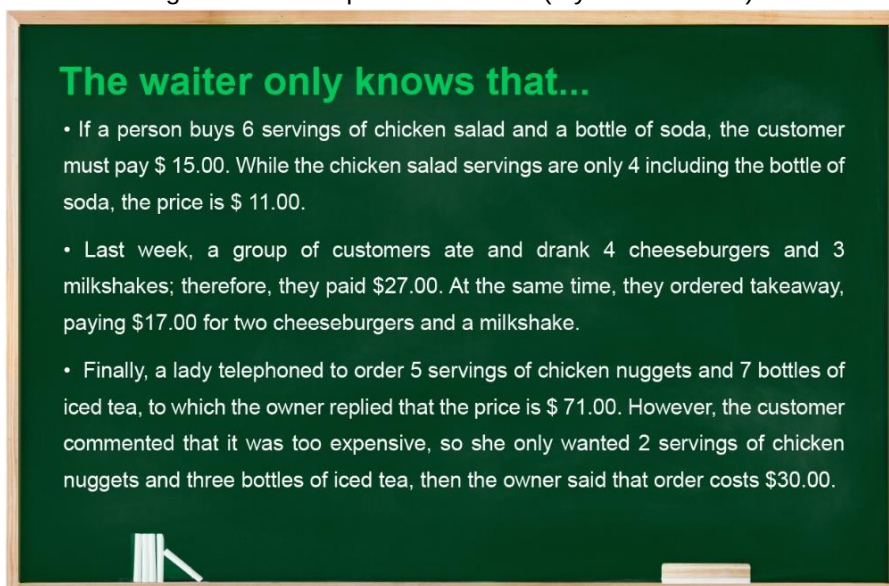


Figure 25: Waiter's information to determine the prices (my own creation).

DINE

FAST FOOD

CHEESEBURGER	\$
CHICKEN NUGGETS	\$
CHICKEN SALAD	\$

DRINKS

MILKSHAKE	\$
ICED TEA	\$
SODA	\$

Figure 26: Menu (my own creation).



AMOUNT	DISH	PRICE
2	CHEESEBURGER	\$
3	CHICKEN NUGGETS	\$
1	MILKSHAKE	\$
4	SODA	\$
TOTAL		\$

Figure 27: Bill (my own creation).

4.9.8. Session 8: The equation of my life

Topic: The equation of my life		Time: 40 minutes
Aim: To assess maths content and language acquired based on self-reflection.		
TEACHING OBJECTIVES		
Content	Cognition	Culture
Linear equations and simultaneous equations review.	Reflecting on the content knowledge learnt and the language acquired.	Sharing ideas and beliefs with the class.
Communication		
Language of learning	Language for learning	Language through learning
Algebraic vocabulary. Common expressions.	Asking/answering questions. Language to give opinions. Language to write essays.	Recycling discussion skills. Extent writing skills.
LEARNING OUTCOMES		
Content	Language	
Learners will be able to: Master linear equations, simultaneous equations and problem solving.	Learners will be able to: - Discuss the importance of equations. - Describe the main procedures to solve simultaneous equations. - Interpret text problems.	
PROCEDURE		
Time	Activities	Materials/ resources
10'	Video: How a mathematical equation can lead to a happier life. Discussion: After watching the video, students and the teacher reflect on whether there can be an equation to reach real happiness. Each student writes the equation of his/her life.	Video: https://tinyurl.com/ycklnpsv (2'15'')

10'	Interactive quiz: The world of equations	Educaplay: https://tinyurl.com/y74l5yj7 (Sample 8)
20'	Writing: Self-reflection Students have to write an essay to describe their experience of studying mathematics through English by covering the following points. • Mathematics learning • Language learning • Difficulties • Motivation	Pencil/pen notebook
ASSESSMENT		
• Content is assessed considering the description of the maths contents learnt. • Language is assessed taking into account listening and reading comprehension of maths problems, written production to describe essays, and oral production to discuss a topic.		

Table 15: Session 8 (my own creation).

Sample 8: Interactive quiz

The last session has been planned as an assessment session based on basic maths knowledge, language skills and self-reflection. For that reason, one of the resources used is an interactive quiz created in Educaplay, made up of 10 multiple-choice questions to test maths knowledge and language.

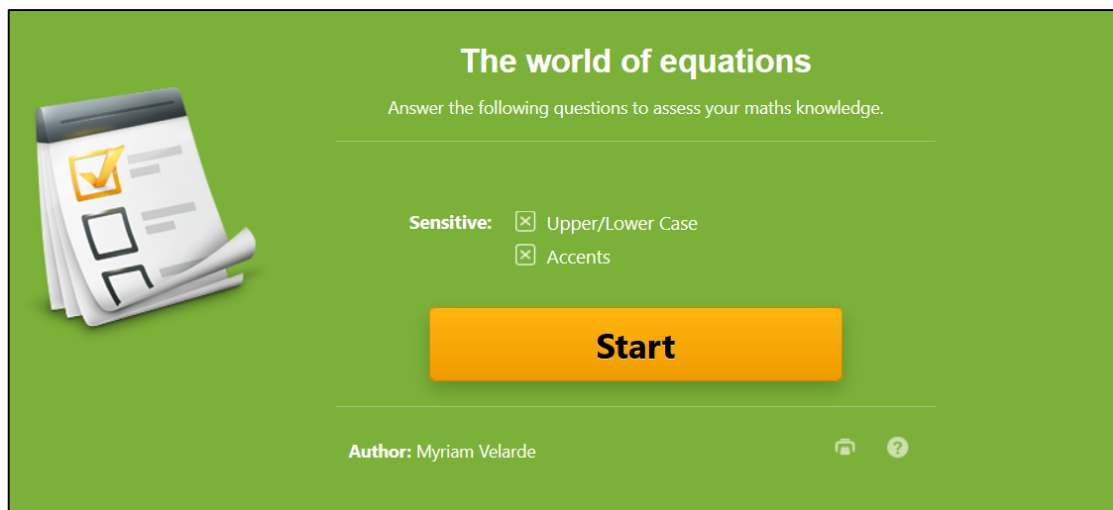


Figure 28: Start screen (my own creation through Educaplay).

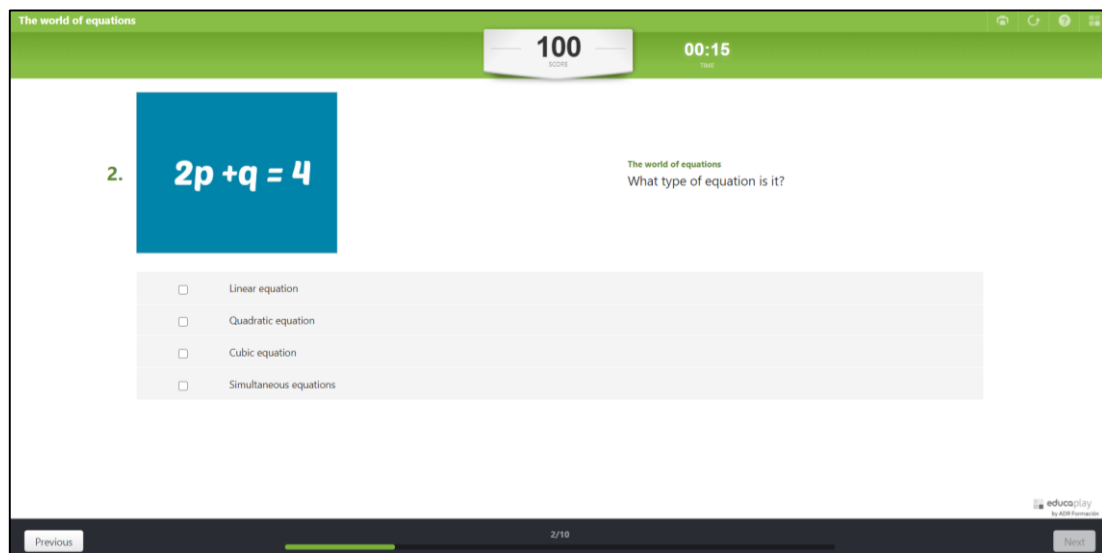


Figure 29: Sample 2 out of 10 questions (my own creation through Educaplay).

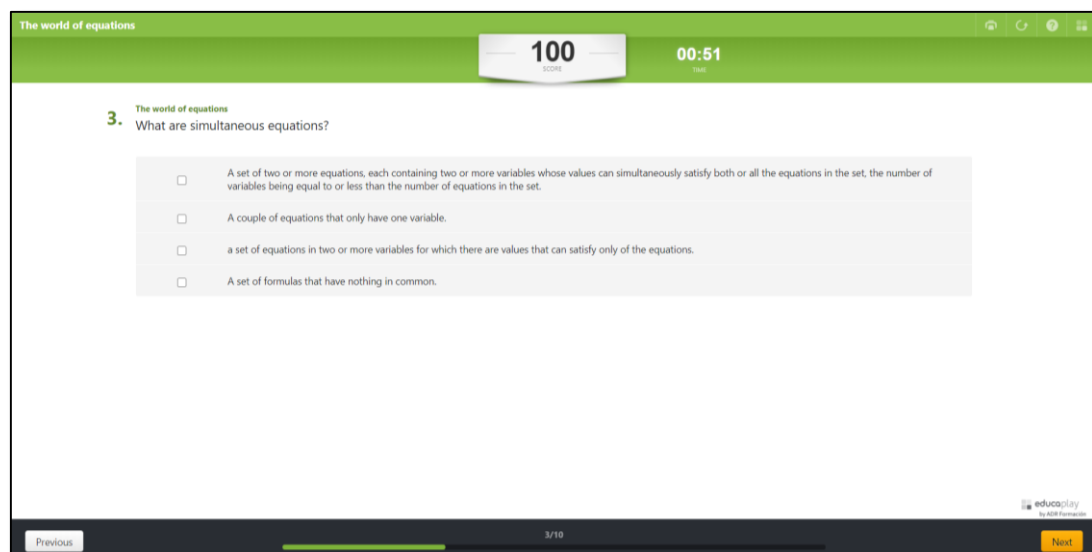


Figure 30: Sample 2 out of 10 questions (my own creation through Educaplay).

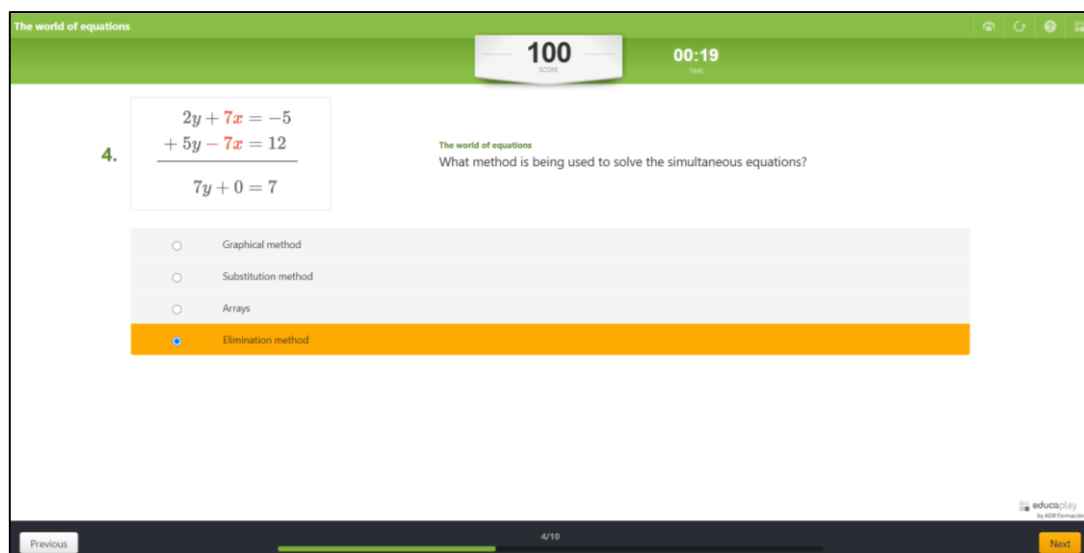


Figure 31: Sample 4 out of 10 questions (my own creation through Educaplay).

4.10. Assessment

Assessment stands for a pivotal stage in education in order to know if learning outcomes are being achieved or not. In CLIL, the question of assessing could be slightly complex given the duality of this approach, but a true fact is that content and language have to be assessed simultaneously and under the same level of importance (University of Cambridge, 2011: 7).

In this CLIL lesson plan, formative and summative assessment is used to determine if learners achieve the expected learning outcomes in accordance with the Ecuadorian curriculum. In order to carry out formative assessment, communicative activities and tasks have been mostly developed putting aside the traditional paradigm of assessing only through tests. Thus, each session mainly comprises formative assessment through performance assessment so that learners can prove their content knowledge and language skills independently and cooperatively (University of Cambridge, 2011: 7). At the end, summative assessment is executed in order to examine the level of contents and language acquired after this CLIL experience.

4.10.1. Content evaluation criteria and evaluation indicators

Based on the key competences determined in the Ecuadorian mathematics curriculum for CSE, the evaluation criteria with their corresponding evaluation indicators covered in this lesson plan are detailed below (my own translation from *Ministerio de Educación Ecuatoriano, 2016: 890-893*).

EVALUATION CRITERIA	EVALUATION INDICATOR
CE.M.4.2. Employing order relationships, algebraic properties of operations on $\mathbb{R}$, and algebraic expressions, to deal with inequalities, equations, and systems of inequalities with solutions of different numerical fields, and solve real-world problems, selecting the notation and the way of calculation appropriate and interpreting and judging the solutions obtained within the context of the problem; analysing the need for the use of technology.	I.M.4.2.4. Learners can solve problems requiring first-degree equations with an unknown in $\mathbb{R}$; use the different notations for the intervals and their graphic representation in the solution of inequalities of the first degree and systems of linear inequalities with two unknowns graphically, in $\mathbb{R}$. (I.1., I.4.)
CE.M.4.3. Defining elementary functions (real function, quadratic function), recognising their representations, properties and formulas algebraic, analysing the importance of axes, units, domain and scales, and solves problems that can be modelled through elementary functions; proposing and solving problems that require the approach of systems of linear equations with two unknowns and quadratic equations; judging the need for the use of technology.	I.M.4.3.5. Learners can pose and solve problems involving systems of two linear equations with two unknowns, quadratic equations, and the application of the root properties of the quadratic equation; judge the validity of the solutions obtained in the context of the problem. (I.4., J.2.)

Table 16: Content evaluation criteria and evaluation indicators. My own translation from *Currículo de los niveles de educación obligatoria* (Ministerio de Educación Ecuatoriano, 2016: 890-893)

It should be emphasized that these evaluation criteria in their entirety must be applied throughout several didactic units, and correspond to the specific topics reviewed in this CLIL lesson plan. The following table shows what content evaluation criteria are present in each session.

EVALUATION CRITERIA	SESSIONS							
	1	2	3	4	5	6	7	8
CE.M.4.2.	x	x	x					x
CE.M.4.3.				x	x	x	x	x

Table 17: Connection between sessions and content evaluation criteria (my own creation).

4.10.2. *Language evaluation criteria and evaluation indicators*

Bearing in mind the key language competences developed in this CLIL lesson plan as per the Ecuadorian EFL curriculum for CSE, the evaluation criteria and their respective evaluation indicators are detailed below (*Ministerio de Educación Ecuatoriano*, 2016: 38-65).

EVALUATION CRITERIA	EVALUATION INDICATOR
CE.EFL.4.4. Demonstrating the ability to ask for and give information and assistance using appropriate language and interaction styles in a variety of social interactions.	I.EFL.4.4.1. Learners can demonstrate an ability to give and ask for information and assistance using level-appropriate language and interaction styles in online or face-to-face social and classroom interactions. (J.2, J.3, J.4, I.3)
CE.EFL.4.6. Listening for Meaning: Understanding and following the main idea in spoken texts set in familiar everyday contexts, provided speech is clear and articulate, and deducing the meanings of unfamiliar words and phrases using context clues and/or prior knowledge.	I.EFL.4.6.1. Learners can grasp the general meaning of spoken texts set in familiar everyday contexts and infer changes in the topic of discussion, as well as deduce the meanings of unfamiliar words and exchanges through the use of context clues, provided speech is given slowly and clearly and there is sufficient visual support. (I.3, S.1, J.4)

CE.EFL.4.7. Listening for Information: Following and identifying some main ideas and details in short and straightforward spoken or audio texts set in familiar contexts, when delivered slowly and with visuals to provide contextual support. Using spoken contributions in class as models for one's own speech.	I.EFL.4.7.1. Learners can identify the main idea and some details in short straightforward spoken audio texts set in familiar contexts when the message is delivered slowly and there is other contextual support. (Example: rules for a game, classroom instructions, a dialogue in a scene from a cartoon or movie, etc.) Learners can use other classmate's contributions in class as models for their own. (I.2, I.3, S.4)
CE.EFL.4.10. Interaction – Interpersonal: Participating effectively in familiar and predictable conversational exchanges by asking and answering follow-up questions, provided there are opportunities to use repair strategies (e.g. asking for clarification) and sustain conversational exchanges in pairs to complete a task, satisfying a need or handle a simple transaction.	I.EFL.4.10.1. Learners can effectively participate in familiar and predictable everyday conversational exchanges in order to complete a task, satisfy a need or handle a simple transaction, using a range of repair strategies. (Example: asking for clarification, etc.) (I.3, J.3, J.4)
CE.EFL.4.11. Demonstrating comprehension of main ideas and some details in short simple texts on familiar subjects, making use of contextual clues to identify relevant information in a text.	I.EFL.4.11.1. Learners can understand main ideas and some details in short simple online or print texts on familiar subjects, using contextual clues to help identify the most relevant information. (Example: title, illustrations, organization, etc.) (I.2, I.4)

CE.EFL.4.16. Making use of simple learning resources, including those created by one's self, in order to compare and contrast information, and choose appropriate resources according to the value, purpose and audience of each.	I.EFL.4.16.1. Learners can use and make simple learning resources, both online and in print, in order to compare and contrast information. Learners can choose appropriate resources and critically evaluate the information in these resources, according to the value, purpose and audience of each. (I.1, I.3, I.4, J.2, J.4)
CE.EFL.4.17. Showing an ability to convey and organize information through the use of facts and details and by employing various stages of the writing process, while using a range of digital tools to promote and support collaboration, learning and productivity.	I.EFL.4.17.1. Learners can convey and organize information through the use of facts and details and by employing various stages of the writing process, while using a range of digital tools to promote and support collaboration, learning and productivity. (I.1, I.3, S.4, J.2, J.4)

Table 18: Language evaluation criteria and evaluation indicators. Extracted from English as a Foreign Language for Subnivel Superior (*Ministerio de Educación Ecuatoriano*, 2016: 38-65)

The following table shows what language evaluation criteria are present in each session.

EVALUATION CRITERIA	SESSIONS							
	1	2	3	4	5	6	7	8
CE.EFL.4.4.	x		x	x	x	x	x	
CE.EFL.4.6.	x	x	x	x	x	x	x	x
CE.EFL.4.7.	x	x	x	x	x	x	x	x
CE.EFL.4.10.	x	x	x	x	x		x	x
CE.EFL.4.11.	x	x	x		x	x	x	x
CE.EFL.4.16.	x	x	x	x	x	x	x	
CE.EFL.4.17.		x		x			x	x

Table 19: Connection between sessions and language evaluation criteria (my own creation).

4.10.3. Assessment tools

Assessment is carried out mainly through activities such as discussions, games, tasks, written and oral production both individually and cooperatively in order to know students' content and language progress, attitudes, motivation, etc. by means of their ongoing performance.

Each activity fosters the construction of knowledge under teacher's guidance and, at the same time, it is possible to collect data with the purpose of verifying the degree of understanding and provide enough feedback. It is important to let students know all the positive aspects developed during their performance, valuing and respecting their learning style and autonomy.

As mentioned, the assessment of content and language is based on students' performance, and bearing in mind that each specific activity provides useful information, these activities are going to be assessed by using tools such as interactive quizzes, worksheets (detailed in each session), games (detailed in each session), rubrics (for maths problem solving and for oral and written production) and a teacher logbook (behaviour, attitudes, ICT experience, possible conflicts and anecdotes). The rubrics used to assess each activity involving problem solving, oral and written production are showed below.

CRITERIA	EXCELLENT	GOOD	FAIR	POOR
	4	3	2	1
Organization	The work is presented in an clear and well organised manner that is easy to read.	The work is presented in an organised manner that is quite easy to read.	The work is presented in an organised way, but it can be somewhat difficult to read.	The work looks sloppy and disorganised. It is difficult to know what information is related.
Mathematical reasoning	Uses complex and outstanding mathematical reasoning.	Uses effective mathematical reasoning.	Some evidence of mathematical reasoning.	Little or no evidence of mathematical reasoning.
Mathematical concepts	The procedures demonstrate a complete understanding of the mathematical concept used to solve problems.	The procedures demonstrate a good understanding of the mathematical concept used to solve problems.	The procedures demonstrate some understanding of the mathematical concept necessary to solve problems.	The procedures demonstrate a very limited understanding of the underlying concepts required to solve problems.
Interest and motivation	Is completely motivated and interested in the subject and strive to do things as well as possible.	Is usually motivated and interested in the subject although does not always strive to do things well.	Is not motivated shows little interest in the subject and does things to comply.	shows no interest in the subject or in doing things well.

Table 20: Rubric to assess solving-problem skills. (Based on Rojas Rojas & Tapia Malla: 2016: 148; Sierra Maldonado, 2010: 1)

CRITERIA	EXCELLENT	GOOD	FAIR	POOR
	4	3	2	1
Task achievement	Responds correctly to the task, adjusting to the topic.	Fits the topic more or less including unnecessary information.	Almost does not fit the topic or does not respond exactly to the task.	The topic is totally different, there is no relationship with the task.
Grammar accuracy	Uses several grammatical tenses correctly. Sentences are well structured and use punctuation marks accurately.	Uses few grammatical tenses but it has some flaws in the structure or verbs.	Uses only a verb tense or there are several mistakes in the formation of sentences or the use of verbs.	Verb tenses are wrong or the structure of most sentences is wrong.
Coherence and cohesion	Organises ideas coherently with an excellent progression and using cohesive devices.	Organises ideas logically with an good progression and using some cohesive devices.	Presents ideas with some organisation but there may be a poor progression, with inaccurate or repetitive cohesive devices.	Presents ideas with no organization and the progression is totally unclear. There are no cohesive devices.
Lexical resource	Varied vocabulary appropriate to their age and current topic.	The vocabulary is little varied although it includes words from the unit. There may be some wrong words.	The vocabulary is very repetitive or there are several words that do not match the sentence.	The vocabulary used is very basic and repetitive.

Table 21: Rubric to assess written production. (Based on IELTS TASK 2 Writing band descriptors (public version); TOEFL iBT Test Integrated Writing Rubrics)

CRITERIA	EXCELLENT	GOOD	FAIR	POOR
	4	3	2	1
Fluency and coherence	Understandable and fluent speech, with little or no hesitation. Total willingness to speak developing the topic coherently.	Clear and flowed speech, although with some hesitation because of the search of words. Willingness to speak trying to develop the topic coherently.	Somewhat unclear speech although some ideas are still comprehensible. Little willingness to speak with some communication problems.	Unclear speech with long pauses and repetitive or inaccurate use of phrases. Reluctance to speak unable to convey basic information.
Lexical resource	Correct use of the vocabulary of the level, communicating in an excellent way. Use a precise and varied vocabulary for the level.	The vocabulary used adapts to the communicative situation although it is occasionally repetitive and makes few lexical errors, but communicating in an acceptable way.	The vocabulary used sometimes adapt to the communicative situation and makes frequent lexical errors, communicating partially. Use limited or repetitive vocabulary for the level.	The vocabulary used does not adapt to the communicative situation and constantly makes lexical errors using an excessively limited and imprecise vocabulary for the level.
Grammar	Shows good grammatical control trying to use complex structures with minor mistakes.	Shows reasonable grammatical control. Make mistakes that do not cause misunderstanding and sometimes correct them.	Uses simple structures correctly, but makes continuous basic mistakes.	Shows insufficient control of simple and basic structures. These errors make the speech unclear.
Pronunciation	Has a clear and intelligible pronunciation. Use correct intonation.	Makes few pronunciation mistakes. The intonation used are generally adequate.	Makes pronunciation mistakes. The intonation used is somewhat inadequate.	Makes a lot of pronunciation mistakes. The intonation is generally inadequate.

Table 22: Rubric to assess oral production. (Based on IELTS SPEAKING band descriptors (public version); TOEFL iBT Test Integrated Speaking Rubrics)

5. CONCLUSIONS

This MA dissertation has been developed to analyse the main features involved in mathematics teaching through CLIL by means of an extensive literature review, to finally propose a lesson plan to introduce CLIL in an Ecuadorian monolingual school for the first time, aiming to improve mathematics and EFL competences as well as students' motivation, given that there is a noticeable lack of interest in studying these subjects, and some academic deficiencies have been detected.

CLIL is an innovative approach implemented in Europe as a solution that could reduce the rates of monolingualism of a country as long as CLIL programmes are correctly executed, but as explained, there are some conflicts at the moment of applying CLIL in schools mainly due to the lack of well-qualified teachers. CLIL is not only to know a subject and a L2, it goes far beyond because of the implication of using proper methodologies, materials and resources. Overall, apart from content and language, there are pedagogical aspects that have to be well founded to succeed in this dual approach.

Many are the non-linguistic subjects that can be taught through a content-language approach as CLIL; however, mathematics teaching in L2 may stand for a real challenge considering that the subject itself is complex to understand even in L1. There are several difficulties that CLIL teachers in charge of mathematics teaching have to face such as the complexity of the subject, the lack of motivation and interest and the perception of difficulty, just to mention some. Furthermore, dealing with a non-native language demands more effort than in mainstream education, where the main concern is content learning, while in CLIL, learning outcomes have to be reflected in content and language simultaneously.

In CLIL, maths teachers have to create a pleasant and confident learning environment due to the fact that changing the language of instruction can cause confusion, fear or worry. Language support and scaffolding techniques play a crucial role bearing in mind that some maths terms that are known in L1 can be seen as unknown in other language; besides, most of the maths terms correspond to L2 advanced vocabulary.

The use of proper methodologies is decisive to succeed in CLIL; therefore, each CLIL lesson should be planned putting learners in the central point of the teaching-learning process. In a CLIL mathematical context, the implementation of concrete methodologies and learning strategies should encompass all the abstraction of the

contents fostering the application of skills such as problem-solving, critical thinking, creativity, attention, reasoning, decision-making, etc. On the other hand, the language focus must always be present whereby students see L2 as the vehicle of learning and benefit from it to acquire language skills through real interactions in social and academic contexts.

Lesson planning must be carefully done taking into account the triangulation of content, language and pedagogy. Teaching in mainstream education is different from teaching in bilingual education, so having to plan a CLIL maths lesson in L2 is complex indeed, and even more when there are a lot of challenges to overcome such as the application of effective methodologies that favour to both content and language or the lack of enough CLIL materials, which requires from teachers having to adapt existing materials or designing original ones. CLIL is certainly time-consuming and quite demanding when there is no experience. Nevertheless, the mastery of its theoretical underpinnings and ongoing training can help practitioners perform successfully in the CLIL classroom.

The Ecuadorian Ministry of Education has incorporated CLIL into the EFL curriculum as one of the effective approaches to developing FL competences, but after carrying out this dissertation and planning CLIL maths lessons for CSE in a monolingual Ecuadorian school, it is evident that the potential of CLIL goes much beyond only focusing on language skills. CLIL provides multiple opportunities to transform education and leave behind traditional approaches that make education seen as an enforced compulsory system. Through this CLIL lesson plan, it is expected that the students of the tenth grade of CSE at San Felipe Neri school can benefit from CLIL and notice a real change in the way they learn and acquire a L2.

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