

Master's Dissertation/
Trabajo Fin de Máster

**A DIDACTIC PROPOSAL FOR THE
SUBJECT OF MATHEMATICS IN
COMPULSORY SECONDARY
EDUCATION: “*ALGEBRA
EVERYWHERE. DO YOU DARE TO
FIND IT?*”**

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Finally, this dissertation is dedicated to my students. I have thought about them during the whole time I was writing it.

ABSTRACT

The main objective of this dissertation is to propose a didactic unit based on CLIL and designed for the subject of Mathematics and for the 2nd year of Compulsory Secondary Education in Spain. This didactic proposal aims to show in a practical way how to implement CLIL and Algebra in the classroom, in order to help students to develop oral expression and comprehension skills in English at the same time as they develop mathematical competence. This dissertation includes the main sections of a didactic unit: justification, contextualization, key competences, objectives, contents, methodology, lesson plan and assessment. In particular, this dissertation pays special attention to two methodological strategies: interactive groups and the Flipped classroom. Furthermore, a wide range of original materials has been designed to cover both subject and language contents.

KEY WORDS: Didactic unit, CLIL, Mathematics, Compulsory Secondary Education, Algebra.

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1. INTRODUCTION

Currently, due to the language immersion advocated by the European Union, as well as to social, cultural and professional demands of the global society, language teaching has become crucial in the field of education. That's why, as a result of a European multicultural society, an acronym – and, above all, a language program - has emerged in order to deal with foreign language deficit: CLIL (Content and Language Integrated Learning). In the last century, the instructional model was used in schools around Europe, which resulted in classrooms where contents were chosen exclusively by the teacher, regardless of students' needs or interests. The traditional teacher acted as an imparter of knowledge and followed a closed curriculum, assuming certain rules of teaching and considering the classroom to be the only place where learning could occur. This former approach to foreign language teaching is no longer used, and the current cross-curricular approach is broader and is imparted through the medium of a foreign language. In fact, the learning of, at least, one foreign language has become a key objective of education in Spain, as can be seen in the Organic Law 8/2013, for the improvement of the quality of education (in Spain, commonly known by its acronym LOMCE).

2. JUSTIFICATION

As indicated by Pérez Cañado (2016: 10) and Lasagabaster and Doiz (2016: 111), although the CLIL program has been put into practice with great eagerness in the first decade of the 21st century, especially in Europe, in recent years this confidence has been declining due to the critical attitude towards CLIL which has been adopted by society. Some teachers, parents and also mass media (such as some newspapers' articles) are against the CLIL programme, which contributes towards the negative image it has in society.

Hence, how do we implement CLIL in the classroom satisfactorily? At the moment, I am working as a Maths teacher in Secondary Education and, from my own experience, I believe that bilingual education hasn't been given the attention it deserves. In other words, this kind of education has been given so little care by governments and it is unattractive to educational community. I think that this is due to

a lack of general interest in creating measures to improve the program from the beginning. Apart from this, there is not, unfortunately, much coordination between CLIL teachers and each one understands and teaches in their own way, without common methodology guidelines to follow.

For all the reasons cited above, one of the objectives of this dissertation is to bring to light that an innovative Mathematics didactic unit is possible in CLIL and that it can be student-centered, i.e., in accordance with students' interests and expectations in order to encourage learners' motivation. Moreover, interactive groups and the flipped classroom are proposed as a methodology of work, which are a constructive way of teaching and learning for several reasons we will see later.

The learning proposal submitted in this dissertation has been created for the subject of Mathematics and for students of 2nd year of Compulsory Secondary Education. The didactic unit is called ***Algebra everywhere. Do you dare to find it?*** Algebra is an interesting topic because it offers different ways to work on it and it can be easily related to many situations of real life, so it can be an opportunity to put in to practice authentic English.

Given this backdrop, should be noted that the didactic proposal is based on the **Royal Legislative Decree 1105/2014** of the Department of Education, Culture and Sport of Spain, which develops the curriculum for Secondary Education and Baccalaureate. In addition, this legislation is concreted by means of the **Order of 14th July 2016**, which develops the curriculum of Compulsory Secondary Education for the Autonomous region of Andalusia.

3. CONTEXTUALIZATION

This didactic proposal is focused on the **Secondary School “Juan López Morillas” in Jódar**, a town with 12.000 inhabitants located inland in the province of Jaén (Andalusia, Spain). The majority of students come from families, with traditional customs and attitudes, whose level of education is not very high. In addition, the population of Jódar lives mainly on the cultivation of the olive tree and other forms of

agriculture. Hence, most of the workers are seasonal farmers and, for this reason, many students move to another autonomous community with their families for a period of time during the school year. For example, many of them move to Navarra for the asparagus season and to Castilla La Mancha for the grape harvest. This makes it especially important to follow methodology guidelines in the CLIL program because of considerable instability of the students within the system.

The CLIL program was first introduced in this school in the 2010/2011 for English language. Bilingualism was not mandatory during the first years of its implementation but rather, it was an option for the students. This led to separate, streamed groups and discrimination between CLIL and non-CLIL students. However, since last year, bilingual education was binding for students who started 1.º ESO (1st year of Compulsory Secondary Education). Currently, 1.º ESO and 2.º ESO are integrated in CLIL, while 3.º ESO and 4.º ESO are streamed into bilingual and non-bilingual groups. On the other hand, it should be noted that Baccalaureate is not included in the CLIL program.

With regard to the educational centre, there are 847 students and it's the only secondary school in the town. The school is involved in several educational programs such as "Programa de centro bilingüe (inglés)", "Escuela TIC 2.0", "Escuela Espacio de Paz", "AulaDjake" and "Forma Joven", among others. In terms of school facilities, the centre has a library, an ICT room, a gym, a lab, a conference hall, outdoor tracks and dedicated classrooms (music, arts and technology classrooms). Moreover, all the classrooms have a digital whiteboard and Internet connection. On the other hand, the centre caters for disabled students because it has several access ramps and lifts.

4. KEY COMPETENCES

As is reflected in the **Order ECD/65/2015 of 21th January 2015**, the European Parliament establishes that all the European member states have to develop key competences in education. These competences must be integral to the learning process of a person in order to help people become active citizens and improve self-development. Propelled by this suggestion, Spain and, in particular, the autonomous

community of Andalusia has defined seven key competences, which are as follows: *Competence in Linguistic Communication (CLC)*; *Mathematical Competence and basic competences in Science and Technology (MCST)*; *Digital Competence (DC)*; *Competence of Learning to Learn (CLL)*; *Social and Civic Competences (SCC)*; *Sense of Initiative and Entrepreneurship (SIE)*; and *Cultural Awareness and Expressions (CAE)*.

It is crucial that key competences be closely related to other curricular elements (objectives, contents and evaluation criteria) because this is established in the Order ECD/65/2015 and it is one of the European Parliament's recommendations. In particular, evaluation criteria have already been linked with key competences in the Order of 14th July 2016, therefore, we can say that key competences are indirectly assessed when we assess with evaluation criteria. In addition, the acquisition of key competences is an interdisciplinary issue due to the fact that all subjects should contribute towards acquiring them when they assess their own evaluation criteria.

Accordingly, we might think that key competences are like boxes which are gradually filled up with skills and abilities throughout the school year. In other words, these boxes are slowly filled up with the competence approach of each subject. The figure below shows an example of how three subjects contribute to the acquisition of the Competence in Linguistic Communication (CLC):

Mathematics

E.C.1.1. To express orally and in a reasoned way the process followed in problem-solving.

CLC, MCST.

English

E.C.3.2. To know how to apply appropriate strategies to understand the general meaning, the essential information, the fundamental ideas and the relevant details of a text. **CLC**,

CLL, SIE.

Technology

E.C.3.3. To describe the classification and applications of materials for technical use.

MCST, CLL, **CLC**

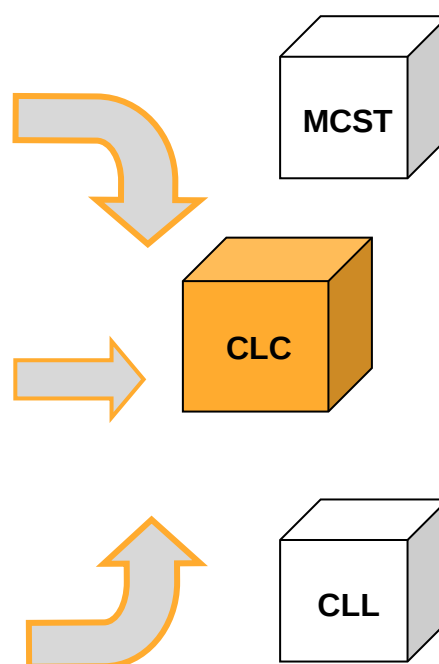


Fig.1. Key Competences' boxes

5. OBJECTIVES

5.1. General objectives

The **general objectives** to be achieved throughout this didactic unit are drafted in the **Order of 14th July 2016**, which develops the curriculum for the Autonomous region of Andalusia:

1. To improve the ability to think reflectively and to incorporate the forms of mathematical expression and reasoning to our language, both in the mathematical processes and in the different areas of human activity.
2. To recognise and to raise situations which can be formulated in mathematical terms, to elaborate different strategies in order to analyse results using adequate resources.
6. To use different technological tools properly (calculator, computer, mobile phone, digital blackboard, etc.) both for doing calculations and for searching, collecting and processing information as well as for supporting the learning process.
8. To develop personal strategies to analyse specific scenarios in order to identify and solve situations, using different resources and instruments and examining new strategies according to the analysis of the results and to their approximate or precise nature.
9. To demonstrate a positive attitude towards problem solving and to show confidence in their capacity to deal with them successfully, acquiring a proper level of self-esteem in order to enjoy creative, manipulative, practical and useful aspects of Mathematics.

5.2. Specific objectives

Aside from the previous general objectives, at the end of this didactic unit the students will be able to attain these **specific aims**:

- **Content aims:**

- a) To translate everyday expressions, model situations or mathematical relationships into algebraic language.

b) To simplify and manipulate algebraic expressions and do basic operations with them.

c) To understand and use the concepts concerning the equations and their elements and use ICT as supporting tools.

d) To use algebra as a tool to model and solve problems and use ICT as supporting tools.

- **Language aims:**

e) To understand word problems in order to build up reading comprehension in a foreign language.

f) To develop oral expression skills, to support and construct arguments to express verbally a problem-solving process.

g) To improve written expression skills and to produce short texts in a foreign language.

- **Other aims:**

h) To develop a respectful and cooperative attitude when in working groups.

i) To develop a spirit of entrepreneurship, self-confidence, critical analysis, the ability to learn how to learn and the decision-making and to assume responsibilities

5.3. Relationship between general objectives, specific objectives and key competences

The relationship between the above-mentioned objectives and the key competences is reflected in this table:

		General objectives					Key Competences
		1	2	6	8	9	
Specific Objectives	a	X	X				MCST
	b	X					MCST
	c	X		X			MCST, DC
	d		X	X	X		MCST, DC
	e	X					CLC
	f	X					, MCST, CLL
	g				X	X	CLC, CLL
	h					X	SCC, SIE, CLL
	i			X	X		SCC, SIE, CLL

Table 1. Relationship between general objectives, specific objectives and key competences

6. CONTENTS

6.1. Subject and language contents

The basic contents of the curriculum were taken from the state and official **Royal Legislative Decree 1105/2014 of 26th December 2014** and from the above-mentioned **Order of 14th July 2016**, which adds the contents specific to Andalusia. The syllabus of Mathematics is divided into 5 content blocks: *Procedures, Methods and Attitudes in Mathematics*, *Numbers and Algebra*, *Geometry*, *Functions* and *Statistics and Probability*. The **conceptual contents of Mathematics** covered in this didactic unit are:

Block 1: Procedures, Methods and Attitudes in Mathematics

- Planning of the resolution process of a problem.
- Mathematical modelling practice, in everyday contexts and in mathematical contexts.
- Confidence in the ability to develop appropriate attitudes and to deal with the difficulties of scientific work.
- Appropriate use of technological resources in the learning process.

Block 2: Numbers and Algebra

- Initiation to algebraic language.
- Translation of colloquial speech expressions of real situations to algebraic language.
- Algebraic language to generalise properties and symbolise relationships. Creation of formulae and general terms based on the observation of patterns. Numerical value of an algebraic expression.
- Operations with simple algebraic expressions. Transformation and equivalences. Identities. Operations with polynomials in simple cases.
- Resolution of first degree equations with one unknown variable. Interpretation of solutions. Problem solving with equations.
- Resolution of second degree equations with one unknown variable.

On the other hand, it is essential to define **language contents** linked with the didactic unit. It should be noted that these contents are not included in any syllabus or official regulation, but they are considered fundamental for the CLIL approach.

Language Contents
• Vocabulary of algebraic expressions' terminology: monomial, polynomial, similar terms, coefficient, literal part, etc. • Vocabulary of equations' terminology: unknown variable, degree, equal sign, like terms, etc. • Comprehension of written short texts or expressions. • Oral expression of ideas and arguments to support or refute a mathematical procedure or a problem-solving process. • Overcoming insecurities and dealing with unknown situations when speaking orally in a foreign language. • A positive attitude to and self-confidence when learning a foreign language. • A respectful attitude towards people who speak in a foreign language.

6.2. BICS and CALP

With regard to language contents, it is not unreasonable to think that there are some differences in acquisition between conversational language and academic language (related to the subject content). In this sense, Cummins (1979) defines two views of language proficiency: BICS (Basic Interpersonal Communication Skills) and CALP (Cognitive Academic Language Proficiency). BICS refers to the everyday, here-and-now language, regardless of the academic aptitude of the person, so it can be called social language or conversational language. Conversely, CALP is closely linked to the language used in academic classes, so it is usually called academic language. According to Gersten *et al.* (2007: 16), CALP "is the language of academic disciplines (science, history, literary analysis), of texts and literature, and of extended reasoned discourse" and "it is more abstract and decontextualized than conversational English."

CLIL seeks the development of the students' target language, through both BICS and CALP so, when planning CLIL lessons, it is crucial to keep in mind both views in order to improve students' performance levels. To clarify this, we develop BICS when giving students instructions, when telling them to set up working groups or when disciplining them. In contrast, we evolve CALP when teaching specialized vocabulary or expressions related to the subject matter. In the session by session development of this didactic unit, the proposed activities are differentiated according to BICS and CALP.

6.3. The 4Cs Framework

Needless to say, in the CLIL classroom there is a close relationship between content learning and language learning. For this reason, Coyle, Hood and Marsh (2010: 41) proposed a conceptual map of CLIL based on the **4Cs Framework (Content, Cognition, Communication and Culture)**:

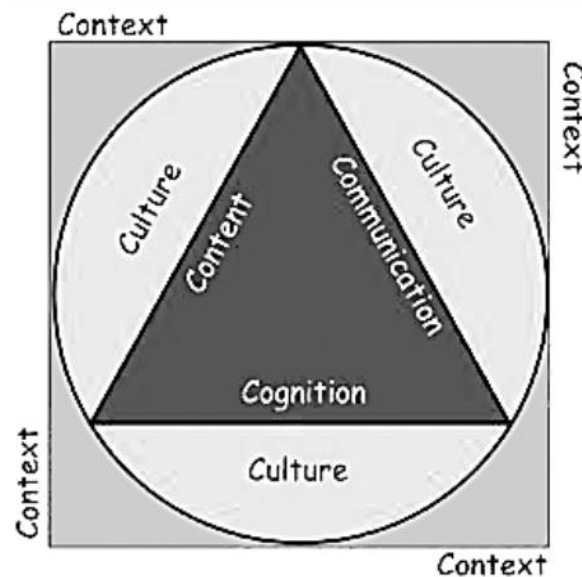


Fig.2. The 4Cs Framework (Coyle, Hood and Marsh 2010: 41)

Here we classify the above-mentioned subject and language contents taking into account the 4Cs Framework:

CONTENT	COGNITION
CT1 - Initiation to algebraic language. CT2 - Operations with simple algebraic expressions. Transformation and equivalences. Identities. Operations with polynomials in simple cases. CT3 - Resolution of first degree equations with one unknown variable. Interpretation of solutions. Problem solving. CT4 - Resolution of second degree equations with one unknown variable. CT5 - Vocabulary of algebraic expressions' terminology: monomial, polynomial, similar terms, coefficient, literal part, etc. CT6 - Vocabulary of equations' terminology: unknown variable, degree, equal sign, like terms, etc.	CG1 - Planning of the resolution process of a problem. CG2 - Mathematical modelling practice, in everyday contexts and in mathematical contexts. CG3 - Translation of colloquial speech expressions of real situations to algebraic language. CG4 - Algebraic language to generalise properties and symbolise relations. Creation of formulae and general terms based on observation of patterns. Numerical value of an algebraic expression. CG5 - Comprehension of written short texts or expressions.
COMMUNICATION	CULTURE
CM1 - Oral expression of ideas and arguments to support or refute a mathematical procedure or a problem-solving process. CM2 - Overcoming insecurities and deal with unknown situations when expressing orally in a foreign language.	CU1 - Appropriate use of technological resources in the learning process. CU2 - Confidence in the ability to develop appropriate attitudes and to deal with the difficulties of scientific work. CU3 - A positive attitude and self-confidence to learn a foreign language. CU4 - A respectful attitude towards people who speak in a foreign language.

Table 2. Subject and language contents in the 4Cs framework

7. METHODOLOGY

7.1. Methodological strategies

As mentioned in the introduction, in the last century, the instructional teaching model was used all over Europe. In this former model of teaching, the learning process was guided only by the teacher, and students were rarely involved in their learning process. Students' needs, aspirations and interests were not taken into account, and "attention to diversity" was an undefined term, although of course measures for treating diversity were urgently required. Ultimately, the necessity for a change of focus was evident, and the 21st century brought immediate challenges to the educational scenario. The roles of both teacher and students were fully revised, giving rise to **student-centered methodologies**. This way of teaching can be defined as:

A perspective that couples a focus on individual learners (their heredity, experiences, perspectives, backgrounds, talents, interests, capacities and need) with a focus on learning (the best available knowledge about learning and how it occurs and about teaching practices that are most effective in promoting the highest levels of motivation, learning, and achievement for all learners.

(McCombs & Whisler, 1997: 9)

In other words, student-centred methodologies embrace different teaching methods which are focused on: students' context and background, what learning outcomes are they able to achieve, what teaching methods are more effective for students and how aware they are of their own progress and learning process. As a result, the teacher's role has changed radically from what it was in the last century and the teacher becomes a facilitator providing guidance and support and analysing what happens in the classroom. To make student-centred methodologies more meaningful, in the following sections we will examine two particular methodological strategies in detail which will then be applied in this didactic unit.

7.1.1. Interactive groups: theoretical approach and implementation

It's clear that an exhaustive analysis of the teaching context and the sociocultural environment of the school is key to improving the teaching-learning process. In my humble opinion, we should analyse the following factors: the main source of livelihood of the neighbourhood, the level of study of the families, the economic status of the families and the unemployment rate, the migration rate, the level of absenteeism, the school dropout rate, the general curricular level, the cultural tradition of the use of a foreign language, the social acceptability and tolerance to diversity, among many other parameters we consider appropriate. In view of the above, teaching in rural areas is often an arduous task because of the students' low socio-economic status. In this sense, we can affirm that sometimes the educational system fails to capture these aspects and many students find themselves excluded from education, leading to behavioural problems and, ultimately, to school dropout.

In response to these challenges, the European Commission developed a research project called *INCLUD-ED: strategies for inclusion and social cohesion in Europe from education*, whose ultimate goal was "to identify which concrete actions contribute to promote success in education and social inclusion throughout the different stages of obligatory education (European Commission 2012: 3). The impact of the *INCLUD-ED* project has been so high that some of its research outcomes are reflected in five European recommendations that aim to guide improvements in education (Flecha 2015: 4).

Among the different inclusion actions that the project proposes, we will focus on interactive groups, which includes cooperative learning communities. As Elboj & Niemelä (2010) state, students' learning processes cannot be separated from the social and cultural context in which they develop relationships with other people, and therefore Elboj & Niemelä (2010) suggested organising classrooms into sub-communities of mutual learners. We will implement an interactive group methodology due to the rural character of the Secondary School "IES Juan López Morillas" for which the didactic unit has been designed.

Interactive group work will be set up as follows: students are divided in heterogeneous groups of 4-6 pupils keeping in mind their abilities or their strengths and weaknesses. Each group is coordinated by an external volunteer, such as parents, former students or social workers from the town. These volunteers do not need to know the content of the subject but rather they only are responsible for ensuring the proper functioning of the group. In other words, volunteers make sure that students are all working and giving ideas, as well as taking care of the material, so anyone can be a volunteer.



Fig.3. Interactive groups. Each group is coordinated by an adult volunteer (Flecha 2015: 33)

The classroom is divided into as many zones as there are groups. In each zone students have to do a different activity, which is supported by an adult volunteer. Teachers are responsible for designing all the activities and briefly explaining to volunteers before each class the activity that they will lead. Each activity takes around 15 or 20 minutes and it is centred on both curricular content and language learning. After completing one activity, the groups rotate to work on a different activity with a different volunteer. As a result of this dynamic, in one hour all the students have worked on four different curricular activities and have interacted with different adults as well as their peers (figure 4):

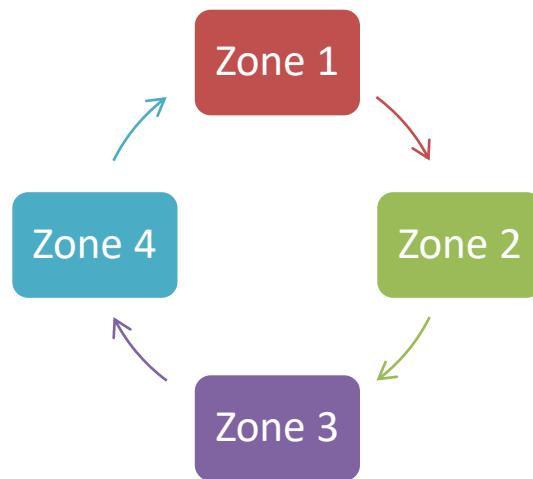


Fig.4. Interactive groups. Each group rotates to work on a different activity with a different volunteer.

According to Flecha (2015), interactive groups is an interesting methodology for three main reasons: teachers can give better attention to the students because they do not need to worry so much about the proper functioning of the each group; the groups provide opportunities for mutual help among students with different learning levels and paces; and the adult volunteers, who are not employed at the school, can see how students work and improve self-development at the school. In addition, from my own experience, I would like to add that students work in a committed way because they usually know the volunteers (quite possibly they are their parents) so this is a key factor that contributes towards the successful involvement of rural students in the classroom.

Furthermore, as Oliver & Gatt (2010) claim, interactive groups allow more individualized assessment and support. Moreover, it enables students to have diverse sources of support: teachers, volunteers and classmates. Relating to the latter, students with learning difficulties can be helped by more advanced students, and advanced students can identify their knowledge gaps and deepen their content learning when they explain concepts and procedures to their classmates. Ultimately, interactive groups promote solidarity among students.

The number of schools implementing learning communities has greatly increased in recent years. In fact, *INCLUD-ED* research states that absenteeism has been reduced to 20% since the implementation of cooperative and inclusive methodologies

(Flecha 2015: 73). Hence, a possible conclusion could be drawn from this, that interactive groups are a promising tool for reducing school dropout rates and avoiding both educational and social exclusion. It may, therefore, be a useful strategy to use in this didactic unit.

7.1.2. Flipped Classroom

As the technological movement has been emerged in recent years, a new teaching method has become a trend in education: the *Flipped Classroom*, also known as “*Clase Invertida*” in Spanish. There is a lack of consensus on what exactly the Flipped Classroom is, and the literature about it is very extensive. However, we will attempt a composite definition of the Flipped Classroom close to the most commonly accepted definition, with reference to how it will be used in this didactic unit. As its name suggests, in the Flipped Classroom the concepts of “learn contents in class” and “do activities at home” are inverted or reversed. In other words, we can define the Flipped Classroom as “an educational strategy that consists of two parts: firstly, students watch a didactic video at home, making notes or a conceptual map. Then, students come to class with some background knowledge and put the content into practice by doing activities”. Hence, the Flipped Classroom involves a transformation of the teacher’s role. In a traditional lecture class, teacher presents content in a way to engage students and catch their attention, but Flipped Classroom replaces the “informant teacher” with audio-visual content and replaces homework with activities in the classroom in which the teacher assumes the role of guide (figure 5):

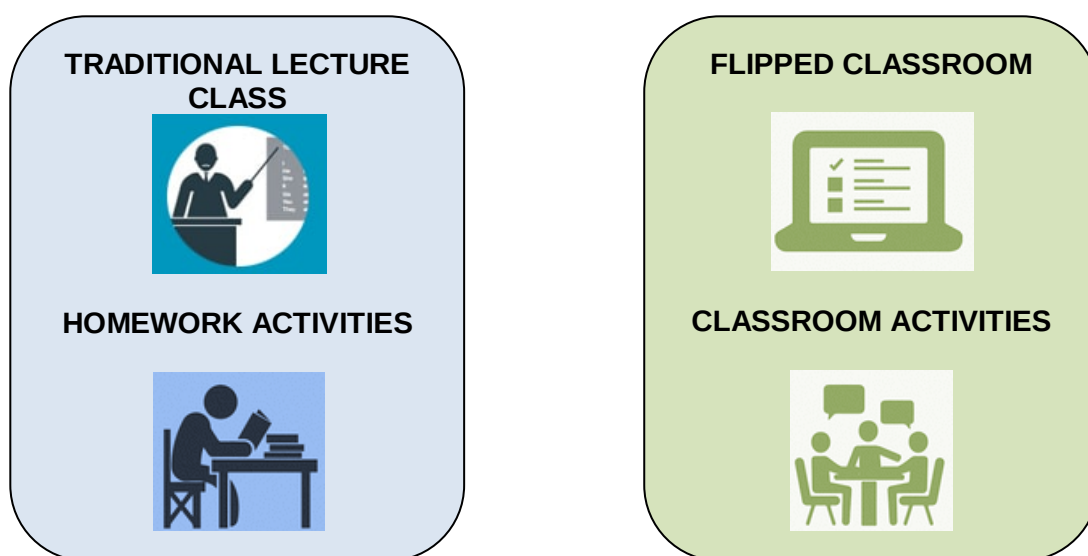


Fig.5. Comparison between traditional lecture class and Flipped Classroom.

The most meaningful learning in a Flipped Classroom occurs as a result of efficient use of the class time (Tucker 2012: 82-83). In fact, this better use of time creates a classroom environment which provides constructivist learning and scaffolding because different levels of support are given at different moments of learning. During the video display students shape previous ideas of the content, fostering critical thinking, and go back to class armed with questions. During class time students engage in discussions, activities, problem solving and group work and are supported by the teacher. Furthermore, it should be noted that several research papers have shown that students prefer shorter videos than longer ones (Lowell Bishop, J. & Verleger, M. 2013). Admittedly, long videos might be a bit boring and students could lose their train of thought and not fully understand the content.

It goes without saying that opinions in the education community regarding the Flipped Classroom are mixed. With regard to this, Szparagowski (2014: 2-3) indicates that there are many misconceptions about what the Flipped Classroom actually is, among which the following stand out: students spend too much time in front of a computer screen, videos replace the teacher, students work in a disorganised way and in isolation, or that the Flipped Classroom is just an online course.

For this didactic unit, we consider the Flipped Classroom as a method to be implemented in the classroom. We do not assume this methodology is a unique and magical way of working but, used in parallel with other methods and without forgetting the possible drawbacks, it can be an advantageous tool.

7.2. Didactic materials

Many authors and research papers point out the importance that materials play in the learning-teaching process (Cunningsworth 1984 & Richards 2006) due to their potential as source and resource for learners and teachers. As Mehisto (2012:15) states, “learning materials can be defined as information and knowledge that are represented in a variety of media and formats, and that support the achievement of intended learning outcomes.” Therefore, materials are anything that facilitate the acquisition of knowledge and they can be given in several formats, styles and

mediums. In addition, didactic materials foster the creation of relational links between contents and students' lives, and help students understand how learning can be applied in and outside of school (Mehisto 2012: 16).

We must not forget that CLIL means Content and Language Integrated Learning and, as Wolff states, CLIL is "an approach which is content-oriented but at the same time language-sensitive" (Wolff 2005: 17) or, to put it differently, CLIL is a dual-focused teaching and learning approach in which the L1 (first language) and L2 (foreign language) seek to gradually develop students' knowledge of content and language skills. Hence, it is crucial to select, adapt or create materials which include both subject content and language learning.

Nevertheless, we have to keep in mind that CLIL materials could lead to a high cognitive workload for students because they need to assume the additional challenge of learning through a foreign language. For example, in Mathematics, students might have difficulties solving a word-problem and if it is also written in a foreign language, the likelihood of them failing to resolve the problem is increased. In this sense, Tomilson (n.d.: 7) states that one basic principle of materials development is to maximise students' exposure to the foreign language in a way they be engaged both affectively and cognitively in the language experience. By the same token, materials help students to recognise the limitations of their thinking and learning and also help them to understand when they need extra support (Mehisto 2012: 16).

Consequently, CLIL materials include an important cognitive and affective challenge, so we should provide students with scaffolding, support techniques and foreign language's thinking skills to help them achieve the learning objectives. The following ten criteria were proposed for Mehisto (2012: 17-25) in order to design CLIL materials of quality for both students and teacher:

Quality CLIL materials should...

1. Make the learning intentions (language, content, learning skills) and the process visible to students.
2. Systematically foster academic language proficiency.
3. Foster learning skills development and learner autonomy.

4. Include self, peer and other types of formative assessment.
5. Help create a safe learning environment.
6. Foster cooperative learning.
7. Seek ways of incorporating authentic language and authentic language use.
8. Foster critical thinking.
9. Foster cognitive fluency through scaffolding of content, language and learning skills development helping student to reach well beyond what they could do on their own.
10. Help to make learning meaningful.

Mehisto (2012: 17-25)

To sum up, it is important to highlight that learning materials should be consistent with objectives and the requirements of the official regulations. Moreover, content (illustrations, texts, videos...) must be presented without bias or stereotypes which promote ethnic, social class, racial or gender prejudice. With all those ideas in mind, I have carefully designed all the materials for this didactic unit, which are included in chapter 8 of the present document.

7.3. New technologies

There is no doubt about the relevance of technology these days, especially among younger generations, so it seems obvious that including technology in the classroom is necessary. Although this is true, the difficulty lies in examining the effectiveness of the use of technology and the positive impact in the students' learning process (OECD 2016). It basically means that we should not use a technical tool in the classroom simply because it is attractive and engaging. We should not be distracted from the main objectives and expected learning outcomes. Another key point to bear in mind is that teachers must have well-developed digital competence and they must test any tools, programs and applications before implementing them in the classroom. In the field of education, researchers have classified new technologies as Information and Communication Technologies (ICT) and Learning Technologies Knowledge (LTK). The former refer to all the technological tools available, even if they have not been designed especially for educational purposes; while the latter are all those particular tools which have been created for teaching/learning purposes. In

other words, ICT are tools that provide access to knowledge because they help us to collect, manage and create information, while LTK offer us the chance to implement digital materials in the classroom in order to enhance the teaching-learning process. Hereafter, we enumerate and briefly explain the ICT and LTK we will use in this didactic unit.

a) Tools and online resources

- **Discovery Puzzle Maker:** it is a freely available tool which creates puzzles, crosswords, math squares, hidden messages and so on. It is available on <http://puzzlemaker.discoveryeducation.com/>
- **Math Playground:** it is a free webpage full of games, puzzles and, in general, interactive for learning Mathematics concepts. It can be displayed on any technological device (computer, mobile phone or tablet). All the games and interactive activities are organized by topics and also by educational levels. Available on <https://www.mathplayground.com/>

b) Applications

- **Plickers:** It is a free app that makes it easy for a teacher to get student feedback almost instantly, and the only technology needed is a single classroom computer and a personal mobile phone or tablet. It works as follows: each student is assigned a Plickers card with a unique pattern or code similar to a QR-code, which you can download for free from the Plickers website. The number of each card is the number assigned by the system to each student. Then, you have to manually enter all test questions and multiple choice answers about your subject prior to the class. Each side of the shape has a different letter (A-D) and these letters represent the multiple choice answers:

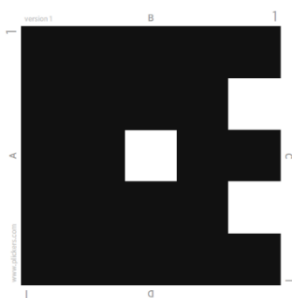


Fig. 6. Plickers' card

Students respond to Plickers questions by holding up their cards and rotating them so that the correct answer is at the top. The teacher scans the cards with the camera on the mobile phone or tablet, scanning all the cards around the classroom. As the system recognizes each card, the results appear instantly on the digital blackboard. In conclusion, Plickers is a free and reliable tool that provides academic performance data in a flash.

- **Kahoot!:** It is a free app which allows gamification in the classroom in the form of a competition. Students need a device with access to the Internet (computer, mobile phone or tablet). It works as follows: a question appears on the digital whiteboard and students have a fixed time to response using a colour key. Automatically, the application shows their answers, so it provides instant feedback. Students can join the game both individually and as a team. Moreover, Kahoot! can be implemented during class time or as homework.

7.4. Cognitive strategies and types of activities

According to the 4Cs framework proposed by Coyle, Hood and Marsh (2010), cognition is one of the basic essentials of this model. Naturally, even before the advent of CLIL, teachers have already helped students to develop their cognition skills. In fact, these thinking skills were classified in 1956 in **Bloom's taxonomy**, which recognized **six cognitive skills' levels**: *Knowledge*, *Comprehension*, *Application*, *Analysis*, *Synthesis* and *Evaluation* (Bloom et al. 1956). However, at the beginning of the 21st century, Anderson and Krathwohl (2001) proposed a revised version of Bloom's taxonomy, changing noun category levels to verb category levels:

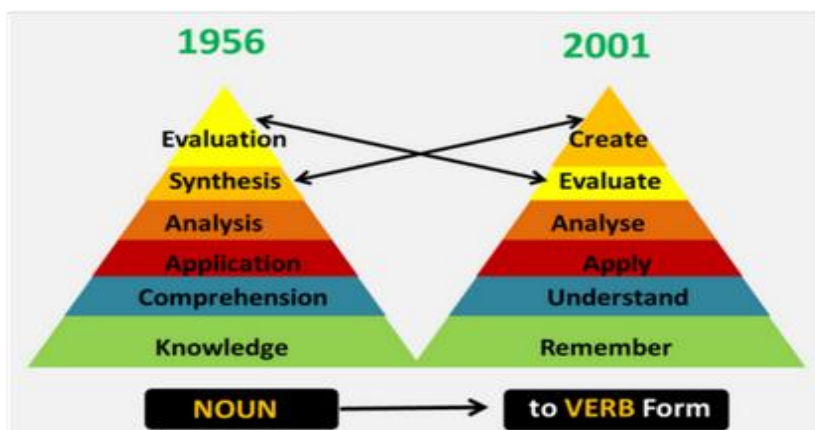


Fig. 7. Bloom's taxonomy: older and revised version. Comparison between both proposals. Retrieved May 20, 2019 from <https://playxipro.com/application-of-blooms-taxonomy-in-e-learning/>

Furthermore, Bloom (1956) classified the 3 lower levels as Low Order Thinking Skills (LOTS), while the 3 upper levels are considered to be High Order Thinking Skills (HOTS). Surely, it is necessary to remember, to understand and to apply a concept before being able to analyse, to evaluate and to create a product with it. In this regard, the type of activities conducted in the classroom are crucial for eliciting high levels of thinking, so teachers must implement different methodologies and activities with the purpose of developing the six cognitive skills' levels. Figure 6 below associates the six different cognitive levels with the different instructions of activities or tasks according to the compilation of Attard, S. *et al.* (2016):

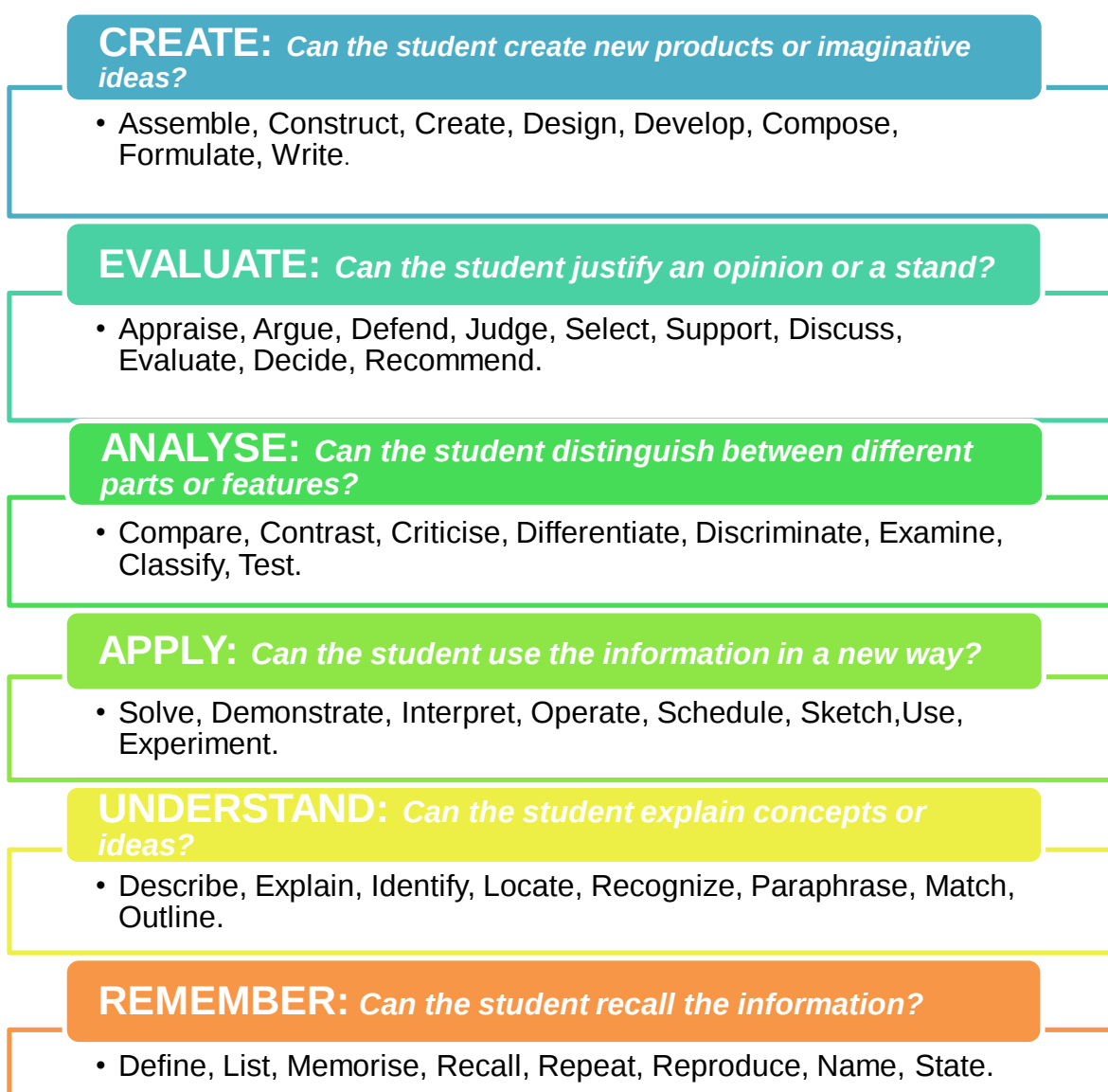


Fig.8. Relation of cognitive levels and activities or tasks¹

¹ Image created through the model of Richard C. Overbaugh and Lynn Schultz.
[<https://www.odu.edu/content/dam/odu/col-dept/teaching-learning/docs/blooms-taxonomy-handout.pdf>]

The activities designed for the didactic unit are classified as LOTS or HOTS depending on the required cognitive skills. Likewise, the didactic unit covers different types of activities depending on the stage of the learning process the students are at. In this sense, there are three types of activities:

a) Introductory activities

Their main purposes are: introduce the topic, generate motivation and interest, examine the previous knowledge of students and acquire basic concepts and ideas about the topic.

b) Developmental activities

The objective of these activities is to apply concepts and procedures in order to develop fluency, get practice and gain self-discipline and confidence.

c) Consolidation activities

Their main goal is to acquire knowledge in a constructive and meaningful way. These activities entail problem-solving discussion, interpretation and critical analysis of information and ability to synthesize content, among others. Therefore, consolidation activities are cognitively highly demanding for learners.

Needless to say, the activities should be introduced progressively and gradually, starting with introductory activities, which serve to catch students' attention. Then, developmental activities can be used to develop abilities, competences and behaviors. Finally, consolidation activities can be used to foster cognitive fluency and critical thinking and make the learning meaningful. At each stage, it would be advisable for students reflect on their own learning so that they develop metacognitive skills.



Fig.9. Types of activities

8. PROCEDURES AND LESSON PLAN

8.1. Initial considerations when planning a CLIL lesson

Designing and planning an effective CLIL lesson is possibly one of the hardest tasks teachers must face. Planning a lesson entails taking into account several aspects and making the following questions clear: *What learning outcomes are pursued? What will students learn? What is the students' prior knowledge? How will students learn? What material will students use? How will the learning emerge? How much time will students need to learn? What are the connections to other topics?*

Although learning can take place at any time and in any place, we will focus on planning the lesson time. In IES Juan López Morillas each lesson lasts 60 minutes and the subject of Mathematics is taught four times a week. In summary, we have a total of 12 hours for this didactic unit. The lesson plan plays an important role in students' learning, so it is crucial to carefully consider some aspects:

a) Creating a rich learning environment

Students learn both subject and language content through interaction with one another and with their teachers. When planning a lesson, teachers have to think about how to introduce opportunities for learners to interact with their teacher and also with their peers, making sure that student and teacher exchange and cooperation are balanced. In this sense, the teacher should plan activities and give instructions that foster social interaction and team work skills.

b) Promoting education in ethic and democratic values

It goes without saying that education in values and rights are essential elements for students to develop a civilised behaviour and to participate in today's society. For this reason, in this didactic unit we will focus on the following **cross-curricular issues**:

- **Fighting homophobia and LGTBIQ-phobia** to show that sexual diversity is completely normal and enriches us as people, thereby also contributing to a plural, diversity and tolerant society.

- **Gender equality in the world of work** to promote gender balance in the job market and eliminate gender stereotypes.
- **Management of domestic economy** in order to increase students' awareness about cost of living and the importance of saving.

c) Adequate selection and adaptation of materials

According to the previous point, the design or adaptation of materials is fundamental because they are the means by which students learn. In addition, they also facilitate the understanding of contents. Materials should be in line with the students' age and educational background, they should foster ethical and civic values and they can come in many formats. In section 7 the proposed materials and resources for the activities for this didactic unit are described. In this sense, it is important to point out that we do not have to select a material or resource just because it seems funny or charming. This means that materials should be consistent with the didactic unit and with the objective pursued.

d) Adapting speech to students' proficiency level

Equally important when planning is to think about the lesson's guiding thread and focus on the L2 approach and oral input. Teachers have to make sure that students understand the foreign language when it is used as a vehicle of communication. When planning a lesson, there are some facets related to the teacher's oral input to bear in mind:

- Give clear instructions and explain activities in steps
- Speak slowly and use pauses
- Use synonyms, simple expressions and paraphrasing
- Articulate clearly and play with voice projection and intonation
- Use body language and gestures

Nevertheless, it's important to highlight that a mixture between English and Spanish can be possible. This means that we must use L2 as much as possible but, sometimes, we can use L1 in order to clarify instructions and contents to the students. For example, when students don't know the meaning of a word, teacher

should not translate it immediately into Spanish but rather give clues or use gestures to help the students infer the meaning of the word.

e) Linking contents with other topics.

Students' motivation, interest and cognitive engagement are signs of meaningful learning. In this respect, Echevarría, Vogt and Short (2007: 241) defined the term building background, which is a strategy based on connecting new concepts with students' personal experiences and past learning. These authors state that when building background, it is essential to detect students' prior knowledge in order to identify misinformation or discover any gaps that need be filled (Echevarría, Vogt and Short 2007: 241). Hence, lesson planning involves looking for possible connections between the content taught and other topics. Without careful planning, students may never appreciate the subject in a meaningful way because they may ignore its applications and its connections with other knowledge areas.

Finally, it goes without saying that planning good, effective lessons requires work and effort and takes time. However, spending time and effort planning is definitely worthwhile in the long term and, although teacher must reflect on their lesson plans as they use them, planning becomes easier as experience develops.

8.2. Lesson plan: timing and sequence of activities

In this section, the **session by session development** of the didactic unit is detailed. Firstly, we show a summary table of the session, and then a sample of the proposed activities with some comments or considerations, if appropriate. It is important to mention that the lesson plan presented is not rigid but rather it can undergo different changes during the teaching process depending on the students' responses.

8.2.1. Session 1: Initiation to algebra

TIME	ACTIVITIES		CONTENT		MATERIALS & RESOURCES
10'	- Video display: “What is algebra?” (2.42 min). - Test prior knowledge: express and exchange ideas orally to reveal the use of algebra in everyday situations.	LOTS	SUBJECT	Initiation to algebraic language.	Video available at: https://www.youtube.com/watch?v=z0OIXIZKfo0
			LANGUAGE	Overcoming insecurities and dealing with unknown situations when speaking orally in a foreign language.	
15'	- Magical algebra: “How can I guess your age and your shoe size?”. Trick implementation and explanation of it.	LOTS	SUBJECT	Initiation to algebraic language.	Sample 1
			LANGUAGE	Overcoming insecurities and dealing with unknown situations when speaking orally in a foreign language.	
35'	- In pairs, complete Worksheet 1: “Algebraic language” . - Add in the foreign language glossary the mathematical expressions used in Worksheet 1. For example: “lowered by”, “3 times...plus...”, etc.	LOTS	SUBJECT	Translation of colloquial speech expressions of real situations to algebraic language.	Worksheet 1
			LANGUAGE	Comprehension of written short texts or expressions.	

Table 3. Session 1

Sample 1: Magical algebra: “How can I guess your age and your shoe size?”

We introduce algebraic language with a magic trick in order to bring to light that we represent an unknown number or quantity by the variable x . First, we will do the trick with the whole class (each student will work individually), and then we will explain the essence of the trick. To do this, we will write the following table on the blackboard:

STEP 1: Making the trick	STEP 2: Explaining the trick
a) Write your shoe size on piece of paper b) Multiply by 5 c) Add 50 d) Multiply by 20 e) Add 1018 f) Subtract your birth year Then, if your result is, for example, 3813, you are 13 years old and your size shoe is 38	a) x b) $5x$ c) $5x + 50$ d) $(5x + 50) \cdot 20$ e) $(5x + 50) \cdot 20 + 1018$ f) $(5x + 50) \cdot 20 + 1018 - 2005$ ↓ common birth year in 2ºESO If we do operations, we get: $100x + 1000 + 1018 - 2005 = \underbrace{100x}_{\substack{\text{2 zeros} = \text{2 place-value} \\ \text{(thousands)}}} + \underbrace{13}_{\text{tenths}}$

Comments:

- Teacher must write the instructions in the left-hand column and make sure that students understand them.
- The final stage of the explanation of the trick could be expressed in L1 language, but the teacher must highlight vocabulary related to the place-value notation system (tenths, thousandths...).
- As a complementary activity, students could create their own trick to guess another feature.

Worksheet 1: Algebraic Language

Work in pairs. Match each sentence with its algebraic expression. You should cross a word. Put the words found in order in the boxes below. Be careful!

There are extra words and extra algebraic expressions!

Any unknown number or quantity	ON	DID	$x - 8$
Your teacher's age minus 8	WHAT	ZERO	$x + 12$
Ana weighs twice as much as her dog	AND	THE	$x - 12$
Your aunt's age plus 12 years	TO	IT	$x - 6$
The number of books in the library divided by 10	SAY	NICE	x
Javi's pencils minus 6	DOES	AT	$3x + 30$
3 times your peer's savings plus 30€	EIGHT?	WHERE	$0.1x + 30$
6 times the daily amount of fiber in your diet	BELT!		$2x + 4$
The cost of the energy bill if the company charges 0.1€ per kilowatt and a flat fee of 30€			$2x$
4 more than twice an unknown number			$30x + 3$
			$6x$
			$x/10$

The first pair who complete the sentence should:

- Run towards the blackboard
- Write the sentence
- Read it out loud and explain its meaning

WHAT	DID	THE	ZERO	SAY	TO	THE	EIGHT?
				NICE	BELT!		

8.2.2. Session 2: Basic operations with monomials and polynomials I

TIME	ACTIVITIES		CONTENT			MATERIALS & RESOURCES
10'	Word search in order to learn vocabulary of basic concepts of algebra.	LOTS	SUBJECT	Initiation to algebraic language.		Worksheet 2
			LANGUAGE	Vocabulary of algebraic expressions' terminology: monomial, polynomial, similar terms, coefficient, literal part, etc.	CALP	
25'	Flipped Classroom Implementation: 4 students will be selected to explain to the rest of their classmates the operations with monomials and polynomials.	HOTS	SUBJECT	Operations with simple algebraic expressions. Transformation and equivalences. Identities. Operations with polynomials in simple cases.		Video available at: https://www.youtube.com/watch?v=z0OIXIZKfo0
			LANGUAGE	Overcoming insecurities and dealing with unknown situations when speaking orally in a foreign language.	BICS & CALP	
25'	Work in pairs: Fast Calculation Challenge in order to practice algebraic operations.	LOTS	SUBJECT	Operations with simple algebraic expressions. Transformation and equivalences. Identities. Operations with polynomials in simple cases.		Sample 2
			LANGUAGE	A positive attitude to and self-confidence when learning a foreign language.	CALP	

Table 4. Session 2

Worksheet 2: Word search

BASIC VOCABULARY OF ALGEBRA

P	Q	Z	G	G	P	Z	A	V	E
O	L	R	G	M	Z	X	H	X	E
L	O	K	E	H	I	C	P	F	R
Y	A	A	Y	A	H	A	H	P	G
N	X	L	I	J	N	Z	Z	S	E
O	V	A	U	D	U	U	N	U	D
M	L	A	I	M	O	N	O	M	M
I	K	N	I	T	R	S	W	R	L
A	G	A	C	Q	T	O	E	N	R
L	S	K	K	T	N	T	F	Y	W

Complete the **missing words** and find them in the word search:

- 1) A **T__M** consists of numbers and letters multiplied together.
- 2) A **M____IAL** is an algebraic expression that has just one term.
- 3) In a monomial, the number is called **C_____ENT**.
- 4) A **FO____A** describes the relationship between two or more variables.
- 5) The sum of monomials is a **PO____IAL**.
- 6) If you multiply all the terms inside the brackets by the term outside,
then you are **E____DING** brackets.
- 7) The sum of the exponents of every variable is the **D_GR__**.

Sample 2: Fast Calculation Challenge

Instructions:

- Each pair receives 2 templates: one containing algebraic operations and another empty template (see below).
- The teacher, provided with a stopwatch, says randomly a letter and a number, and students must do the algebraic operation of that cell.
- After 20 seconds, the teacher will say another letter and number. This process is repeated until complete the template.
- After completing the whole table, it is time to correct it. To do this, each pair gives the template to the pair sitting behind them. The winning pair gain a prize.

	A	B	C	D
1	$(2x + 2) - (5x - 1)$	$(-30x^4y) : (6xy)$	$\frac{1}{2}x + \frac{5}{6}x$	$(8y) \cdot (-7y^2)$
2	$3 \cdot (-2x)$	$(-3x - 1) - (4x - 1)$	$(-45x^6) : (5x)$	$-5x + 17x$
3	$(-60x^4) : (12x)$	$(11x - 5) + (-3x - 1)$	$(-3x^4y^3) \cdot (-6xy^2)$	$(8xy^3) \cdot (-3xy^2z)$
4	$(-2x^2) \cdot (-3x^4)$	$-\frac{1}{2}x + \frac{4}{3}x$	$(-20x^2) \cdot (-5x)$	$-(10x - 2) + (4x - 5)$
5	$4 \cdot \left(-\frac{5}{4}x\right)$	$(-35x^4) : (7x^4)$	$(-2x + 1) - (x - 3x)$	$-4x - 7x$

	A	B	C	D
1				
2				
3				
4				
5				

8.2.3. Session 3: Basic operations with monomials and polynomials II

TIME	ACTIVITIES	CONTENT				MATERIALS & RESOURCES
This session is held in the ICT room						
25'	Work individually: gamification with Plickers to work out algebraic identities.	LOTS	SUBJECT	Operations with simple algebraic expressions. Transformation and equivalences. Identities. Operations with polynomials in simple cases.		Sample 3
			LANGUAGE	A positive attitude to and self-confidence when learning a foreign language.	BICS & CALP	
15'	Work in groups of 4: Interactive game “Jumping Aliens” to practice operations with monomials and polynomials.	LOTS	SUBJECT	Operations with simple algebraic expressions. Transformation and equivalences. Identities. Operations with polynomials in simple cases		Sample 4 Game available at https://www.mathplayground.com/ASB_JumpingAliens.html
			LANGUAGE	A positive attitude to and self-confidence when learning a foreign language.	BICS & CALP	
20'	Work in pairs (one computer for every two students): basic operations review with Kahoot!	LOTS	SUBJECT	Algebraic language to generalise properties and symbolise relationships. Creation of formulae and general terms based on the observation of patterns. Numerical value of an algebraic expression.		Sample 5
			LANGUAGE	Comprehension of written short texts or expressions.	BICS & CALP	

Table 5. Session 3

Sample 3: Plickers

A multiple-choice question appears on the digital whiteboard:

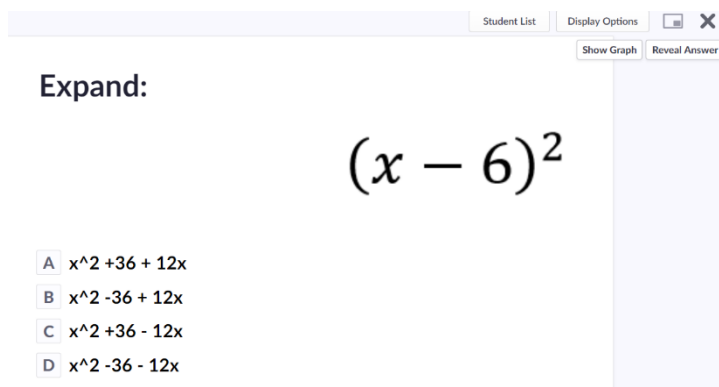


Fig.10. Plickers' question I

Students answer the question by holding up their cards and teacher scan them. Their answers are corrected in real time on the digital whiteboard:



Fig.11. Students holding Plickers' cards

Finally, Plickers is used to give a report of both group and individual average:

Name ^	Total									
Class Average	83%	100%	81%	91%	96%	0%	83%	100%	96%	36%
Betuel	86%	-	C	D	D	-	A	A	C	C
Carlos G	86%	-	C	D	D	-	A	A	C	C
Daniel	86%	-	D	D	D	-	A	A	C	A
Francisco Jesús	71%	-	C	D	D	-	A	A	B	C
Gonzalo	71%	-	B	D	D	-	C	A	C	A
Inma	86%	-	C	D	D	-	A	A	C	D
Iván	100%	-	C	D	D	-	A	A	C	A
Javier	86%	-	C	D	D	-	A	A	C	C
José	100%	-	C	D	D	-	A	A	C	A
Juan Manuel	71%	-	D	D	D	-	B	A	C	A
Lorena	86%	-	C	D	D	-	A	A	C	C

Fig.12. Plickers' report

Sample 4: Interactive game “Jumping Aliens”

The aim of the game is to practice multiplications of monomials:



Fig.13. “Jumping Aliens” I

Up to 4 students can join the game:

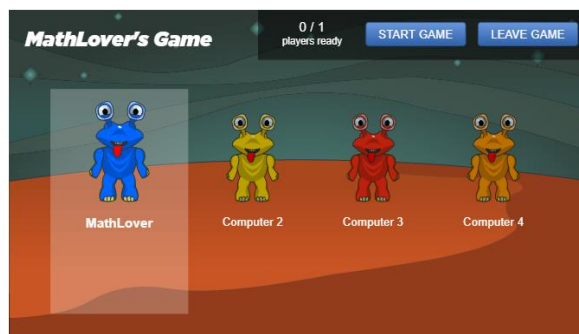


Fig.14. “Jumping Aliens” II

The game consists of a race between the 4 players, who have to multiply algebraic expressions to move forward:

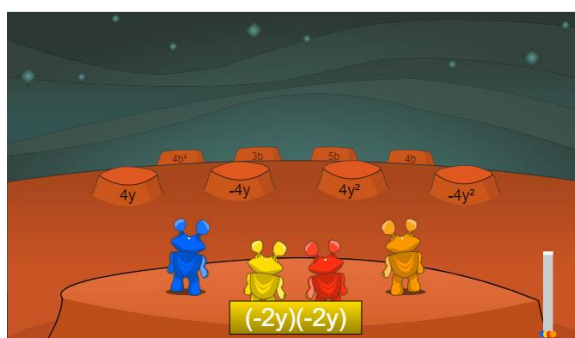


Fig.15. “Jumping Aliens” III

The player who multiplies faster will win the outer space race:

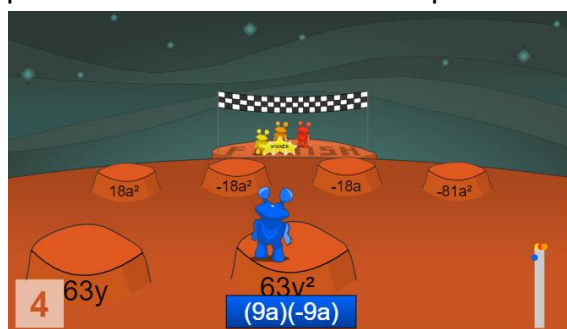


Fig.16. “Jumping Aliens” IV

Sample 5: Kahoot!

All students join the game in pairs by entering a key:



Fig.17. Kahoot's start screen

A question appears on the digital whiteboard:

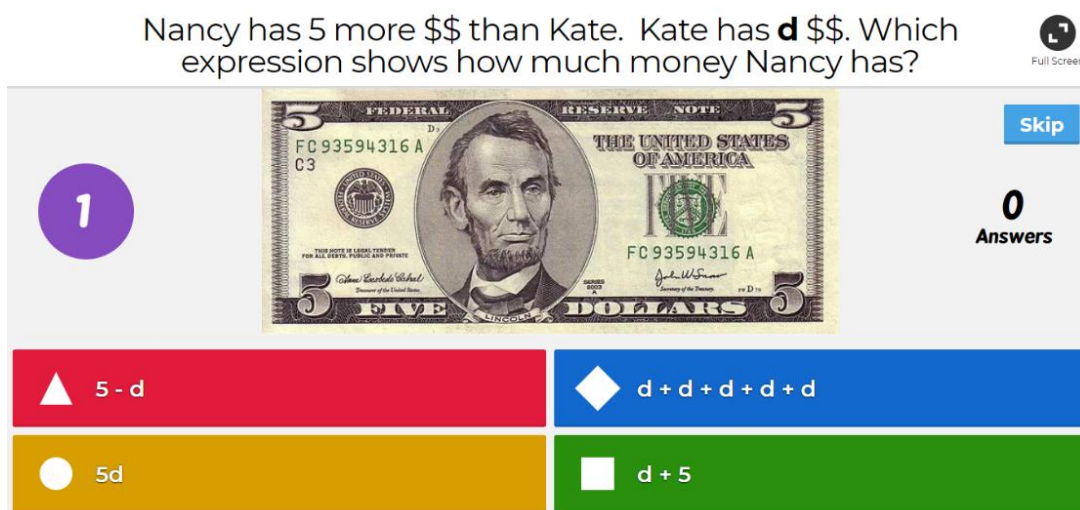


Fig.18. Kahoot's question

Students use the colour key to answer the question and, automatically, the application shows their responses. At this point, teacher can take a few minutes to comment on the activity and to give feedback.

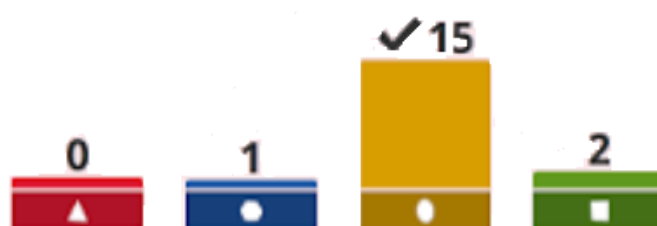


Fig.19. Students' responses summary

8.2.4. Session 4: Review of contents with Interactive Groups I

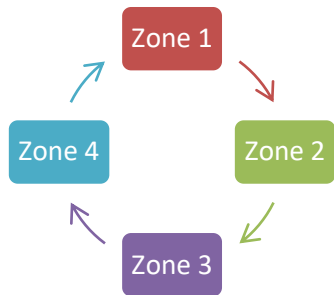
TIME	ACTIVITIES		CONTENT			MATERIALS & RESOURCES
60'	Interactive groups: 	HOTS	SUBJECT	Translation of colloquial speech expressions of real situations to algebraic language.		Zones 1, 2, 3 & 4
				Algebraic language to generalise properties and symbolise relationships. Creation of formulae and general terms based on the observation of patterns. Numerical value of an algebraic expression.		
				Operations with simple algebraic expressions. Transformation and equivalences. Identities. Operations with polynomials in simple cases.		
			LANGUAGE	Comprehension of written short texts or expressions.	BICS	
				Oral expression of ideas and arguments to support or refute a mathematical procedure or a problem-solving process.	BICS	
				A positive attitude to and self-confidence when learning a foreign language.	BICS	

Table 6. Session 4

Zone 1: Birthday's cards

The adult volunteer deals a card to each student like these:

My name is Carlos and today is my birthday	I'm Ana and my birthday is the day before Carlos' birthday	I'm Isabel and my birthday is ten days after Carlos' birthday	I'm Andrés and my birthday is a week after Ana's birthday
--	---	--	--

In the middle of the table, there are several cards with algebraic expressions:

x	$x+7$	$x-10$
$x+7$	$x+6$	$x+1$

Students should talk to each other and match their given card with one algebraic card. The volunteer ensures that students converse in English with one another:

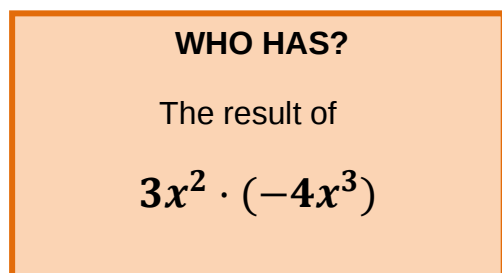


Fig.20. Birthday's cards

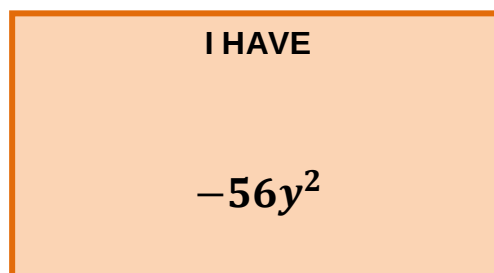
Zone 2: “Who has...? I have...”

The volunteer deals 4 cards to each student. Each card has an algebraic operation on one side, and a result on the back. However, the operation and the result on each card do not correspond to each other. A card looks like this:

Front side:



Reverse side:



A student starts the game saying “*Who has the result of...?*”, and the rest of students have to look at the reverse side of their cards to determine if they have the result. The student who states he/she has the result, should read the front side of the card and then lay it down. The first student to get rid of his/her cards wins.

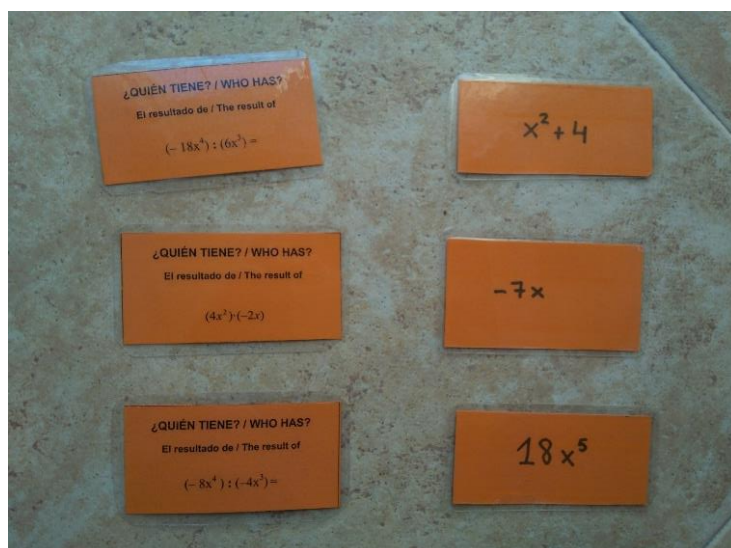


Fig.21. “Who has...? I Have...” cards

In this zone, the adult volunteer is responsible for speeding up the dynamic of the game. Moreover, the volunteer should make sure that all students read their cards in English.

Zone 3: Pentomino Puzzle

Students are provided with 12 coloured wooden pieces, which are labelled using algebraic identities:

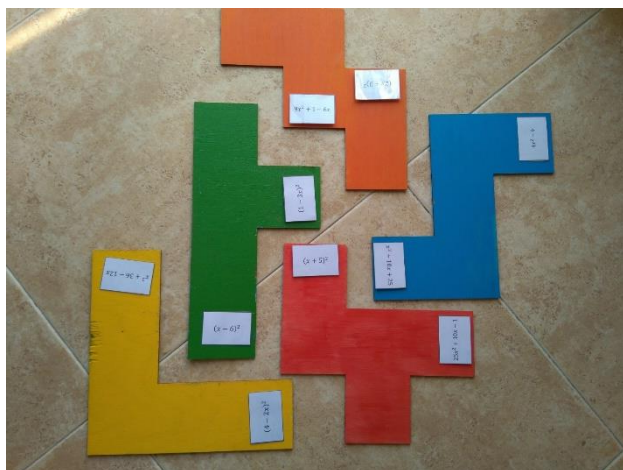


Fig.22. Pentomino. Wooden pieces

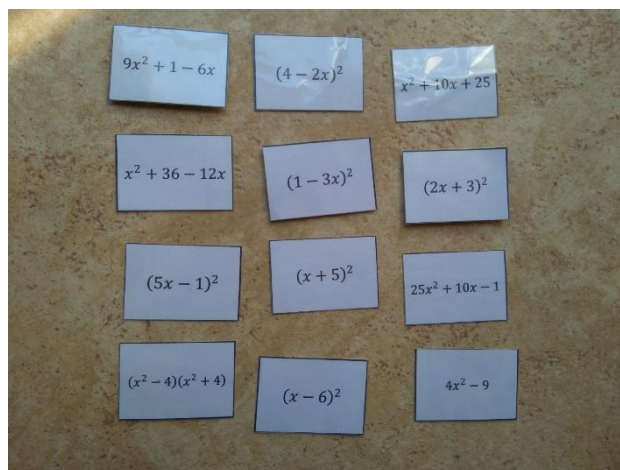


Fig.23. Pentomino. Labels

Students have to match the corresponding algebraic identities in order to form the shape of an animal. In this case, students should get a crocodile:

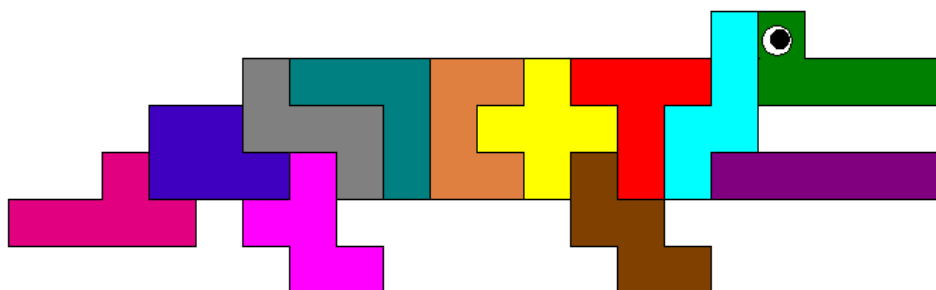


Fig.24. Pentomino. Crocodile

The final result would be:



Fig.25. Pentomino. Final result

Zone 4: Bingo

Students have to complete the above bingo template with only nine of these algebraic expressions:

$$2x - 3 \quad 3x \quad \frac{x}{2} \quad x^3 \quad \frac{x}{3} \quad 2x + 1 \quad \frac{x}{10} \quad x^2 - 2$$

$$\frac{2x}{3} - 1 \quad x - 5 \quad 2x + 10 \quad \frac{x}{2} + x \quad x - \frac{x}{3} \quad a - b$$

$$2x + 1 = 21 \quad 2n + 1 \quad A = a \cdot b \quad A = \frac{b \cdot h}{2}$$

A student assumes the role of spokesperson and says, in this order, the following expressions:

1. Area of a triangle
2. A number minus one third of that number
3. Twice a number plus one is equal to twenty-one
4. The difference between a number and five
5. The double of a number and ten
6. Twice a number minus three
7. The triple of a number
8. Two thirds of a number minus one
9. One tenth of a number
10. Half of a number plus this number
11. A number squared minus two
12. Half of a number
13. A number cubed
14. A third of a number
15. Twice a number
16. Twice a number plus two
17. The difference between two different numbers
18. Area of a rectangle

Norms are the classic rules of bingo: the student who crosses out all the cells first wins the game.

8.2.5. Session 5: Problem-solving I

TIME	ACTIVITIES		CONTENT		MATERIALS & RESOURCES
20'	Work in pairs: Worksheet “ Representing situations of real life ” in order to formulate in mathematical terms situations of everyday life.	LOTS	SUBJECT	Mathematical modelling practice, in everyday contexts and in mathematical contexts.	Worksheet 3
			LANGUAGE	Comprehension of written short texts or expressions.	
40'	Work in groups: The class is divided into 4 groups. There are 2 problems to solve, so that each problem will be solved by two different groups in order to compare results. Once the problem has been solved, the groups should make an oral presentation to discuss their findings.	HOTS	SUBJECT	Planning of the resolution process of a problem.	Sample 6 & Sample 7
				Confidence in the ability to develop appropriate attitudes and to deal with the difficulties of scientific work.	
			LANGUAGE	Comprehension of written short texts or expressions.	
				Oral expression of ideas and arguments to support or refute a mathematical procedure or a problem-solving process.	

Table 7. Session 5

Worksheet 3: Algebra word-problems

REPRESENTING SITUATIONS OF REAL LIFE:

ALGEBRAIC LANGUAGE WORD-PROBLEMS

Remember to use **RUCSAC** rule:

Read, Understand, Choose, Solve, Answer and Check

1. **Sophia works as a carpenter and Diana is an electrician** in a home-maintenance company. Determine an algebraic expression to represent the total cost to employ x carpenters who charge 15€ per hour and y electricians who charge 18€ per hour.
2. **Anna, her girlfriend** and some friends are going to play miniature golf. There are p people in the group. Each person pays 3€ for a round and together they spend 11€ on snacks. Write an expression that represents the total amount that Anna and her friends spent.
3. A factory produces x computers in a month. The cost of production depends on the number of computers produced, and it is given by the expression: $150x-20$ euros. On the other hand, each computer is sold for 400€.
 - a) Write an expression to represent the profit of the factory in a month.
 - b) What will the profit be if the factory produces 100 computers?
4. Shona is getting ready to run a half marathon and she trains on a running track, which is a loop that covers a distance of m kilometres. Last week, Shona ran 12 loops each on Monday and Wednesday, 15 loops on Friday, and on Sunday she ran 18 kilometres. Write an expression that represents the total distance that Shona ran last week.

Comments: The main aim of this worksheet is to develop the comprehension of short texts and to practice mathematical modelling. Furthermore, in problems 1 and 2 include cross-curricular issues: problem 1 reveals that male-dominated professions can also be valid for women; and problem 2 shows two girls who are lesbian.

Sample 6: Domestic economy I → Paying the electricity bill

DOMESTIC ECONOMY I

You're helping your parents to choose the cheapest electricity company. There are two companies, ELIC and ESUR, whose prices are:

ELIC
Contracted power: 6,9 kW
Consumption: 0,13092 €/kWh

ESUR
Contracted power: 6,9 kW
Consumption: 0,148811 €/kWh



Fig. 26. "Brain bulb".

Retrieved May 25, 2019 from
<https://www.alamy.es/>

We pay a fixed rate, which is calculated as:

Contracted power · number of days · 0.116968 → this is a fixed value

- a) Why do we pay this fixed rate? Does it depend on the consumption?
- b) Write the algebraic expression of the fixed rate for a year.

The consumption history of your house shows that you and your family use 650 kWh during the 6 months of winter, and 300 kWh during the 6 months of summer.

- c) Complete the table to calculate the total annual cost of each company:

	ELIC	ESUR
Fixed Rate		
Consumption: Winter months		
Consumption: Summer months		
Total		

- d) Then, what company is most convenient to you?

Sample 7: Domestic economy I → Hiring a professional

DOMESTIC ECONOMY I

- A plumber, who offers a home service, charges a fixed price of 20€ for attending a call, plus the cost of the material used, plus 15€ per hour for the work done.
- a) Write the formula for the **invoice amount** (I), based on the **hours worked** (h) and the **cost of the materials** (M).
- When you are going to pay the service bill, the plumber says that **21% VAT** should be added.
- b) Rewrite the previous formula adding VAT.
- c) What does it mean the 3 letters of VAT? Is it mandatory to pay the VAT? Why do we pay VAT?



Fig. 27. Plumber. Retrieved May 25, 2019 from https://es.123rf.com/photo_56216484_mujer-fontanero-que-trabaja-en-calefact%C3%B3n-central-caldera.html

- Observe the image above:
- d) Do you know any plumbers? Is this a man or a woman? Do you think that working as a plumber is a “man’s job”?

Comments: both problems seek to raise awareness of the importance of house management and to think about the cost of living. These problems require high levels of thinking skills (HOTS) because they include both mathematical modelling and analyse of economic concepts like VAT and fixed rates for electricity. Furthermore, the second problem highlights gender equality in the world of work once again.

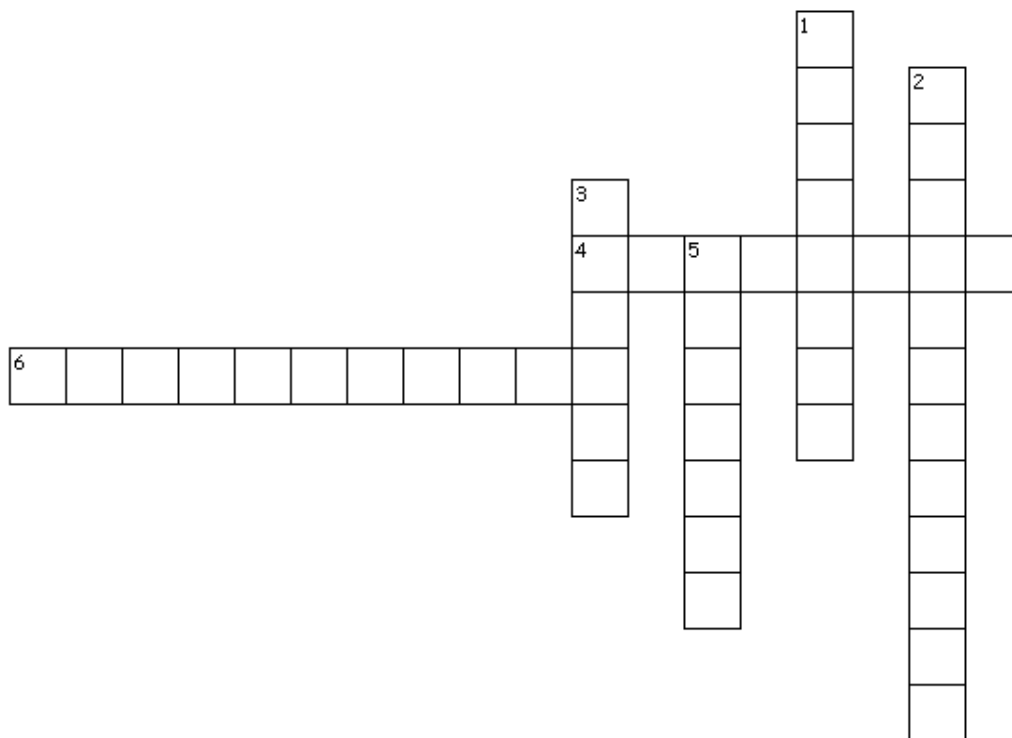
8.2.6. Session 6: First degree equations I

TIME	ACTIVITIES		CONTENT		MATERIALS & RESOURCES
10'	Crossword in order to learn basic vocabulary of equations.	LOTS	SUBJECT	Resolution of first degree equations with one unknown variable. Interpretation of solutions. Problem solving with equations.	Worksheet 4
			LANGUAGE	Vocabulary of equations' terminology: unknown variable, degree, equal sign, like terms, etc.	
30'	Flipped Classroom Implementation: 4 students will be selected to explain to the rest of their classmates the steps to solve a first degree equation.	HOTS	SUBJECT	Resolution of first degree equations with one unknown variable. Interpretation of solutions. Problem solving with equations.	Video available at: https://www.youtube.com/watch?v=fDMxOiS5g7k
			LANGUAGE	Overcoming insecurities and dealing with unknown situations when speaking orally in a foreign language.	
20'	Work individually: "Arm bones" worksheet to remember from the previous course how to solve first degree equations.	LOTS	SUBJECT	Resolution of first degree equations with one unknown variable. Interpretation of solutions. Problem solving with equations.	Worksheet 5
			LANGUAGE	Vocabulary of equations' terminology: unknown variable, degree, equal sign, like terms, etc.	

Table 8. Session 6

Worksheet 4: Crossword

EQUATIONS CROSSWORD



Across

4. Equality of two algebraic expressions that is true for only certain values
6. Left side of the equation

Down

1. If we solve an equation we get the...
2. Right side of the equation
3. Greatest of the terms' degrees
5. Unknown number, which is represented by a letter

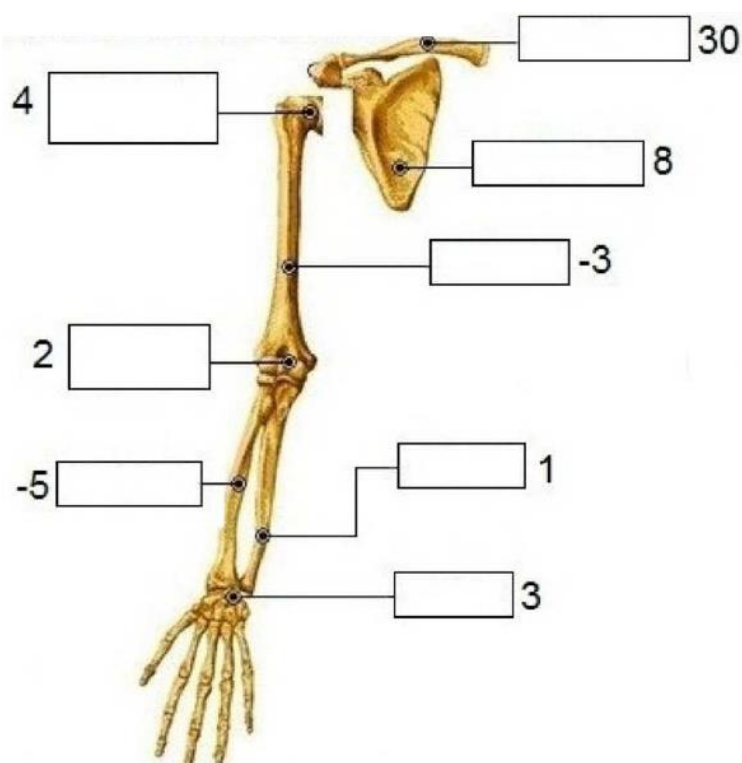
6 of 6 words were placed into the puzzle.

Created by [Puzzlemaker](https://www.discoveryeducation.com) at [DiscoveryEducation.com](https://www.discoveryeducation.com)

Comments: crosswords are a good way to check students' grasp of key concepts and technical terms. This activity is not cognitively challenging but it requires identifying and knowing the basic vocabulary of equations, which is crucial for the learners if they are to make progress in this didactic unit. In addition, in this activity students have to include the unknown words in their glossary of terms in English.

Worksheet 5: Arm bones equations

ARM BONES EQUATIONS



Write the names of the arm bones according to the obtained solutions:

$2x = 6$	$8 - 4x = -3x$	$\frac{x}{6} = \frac{10}{2}$	$2x - 4x = 10$
Carpus	Shoulder Blade	Collarbone	Radius
$6x + 9 = 3x + 12$	$5x = 20$	$2x - 5x = 9$	$10x - 4x = 19 - 7$
Ulna	Shoulder joint	Humerus	Elbow joint

Comments: this is a recall-activity due to the fact that students studied simple first degree equations in the previous academic year. Thus, we add content related to natural sciences bearing in mind that students are studying the bones in another subject.

8.2.7. Session 7: First degree equations II

TIME	ACTIVITIES	CONTENT				MATERIALS & RESOURCES
This session is held in the ICT room						
25'	Work individually: gamification with Plickers to work out first degree equations.	LOTS	SUBJECT	Resolution of first degree equations with one unknown variable. Interpretation of solutions. Problem solving with equations.		Sample 8
			LANGUAGE	A positive attitude to and self-confidence when learning a foreign language.	BICS & CALP	
15'	Work in pairs: Interactive game “Basketball game” to practice two-step first degree equations.	LOTS	SUBJECT	Resolution of first degree equations with one unknown variable. Interpretation of solutions. Problem solving with equations.		Sample 9 Game available at http://www.math- play.com/Two- Step-Equations- Game.html
			LANGUAGE	A positive attitude to and self-confidence when learning a foreign language.	BICS & CALP	
20'	Work individually (one student for each computer): first degree equations review with Kahoot!	LOTS	SUBJECT	Resolution of first degree equations with one unknown variable. Interpretation of solutions. Problem solving with equations.		Sample 10
			LANGUAGE	A positive attitude to and self-confidence when learning a foreign language.	BICS & CALP	

Table 9. Session 7

Sample 8: First degree equations with Plickers

A multiple-choice question appears in the digital blackboard:

Student List

Display Options

Show Graph

Reveal Answer

Solve:

$$\frac{x}{5} + \frac{x}{2} = x - 3$$

A

 $x=1/3$

B

 $x=10$

C

 $x=-10$

D

 $x=-1/3$

Fig.28. Plickers' question II

As in session 3, students answer the question by holding up their cards and the teacher scans them. Their answers are corrected in real time on the digital whiteboard:



Fig.29. Students holding Plickers' cards

Sample 9: Interactive game “Basketball game”

Students work in pairs and they choose between the red team and the blue team:



Fig.30. “Basketball Game”. Start screen.

Students take turns to play. The first one has to solve an equation:

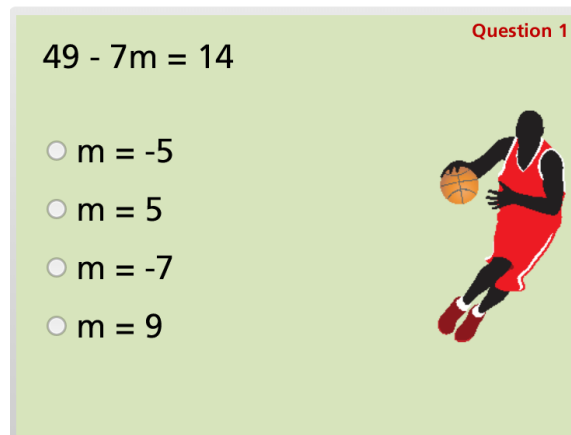


Fig.31. “Basketball Game”. Sample question

If the student chooses the right solution, he/she earns 2 points and the chance for a free throw to get an extra point. The player who scores more points wins the game:

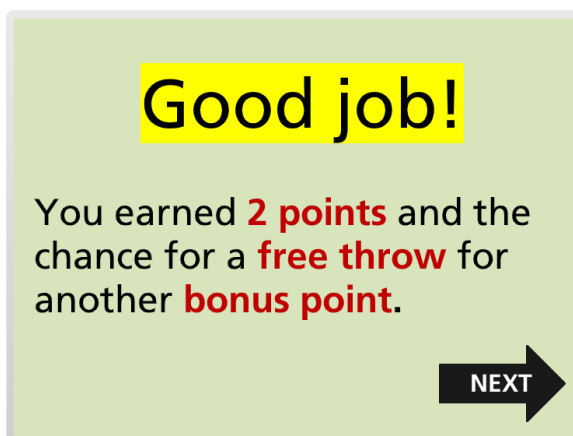


Fig.32. “Basketball Game”. Congratulations message



Fig.33. “Basketball Game”. Score screen

Sample 10: Kahoot!

Once again, students join the game individually and a question appears on the digital whiteboard. In the left hand is the maximum allowed time to answer the question:

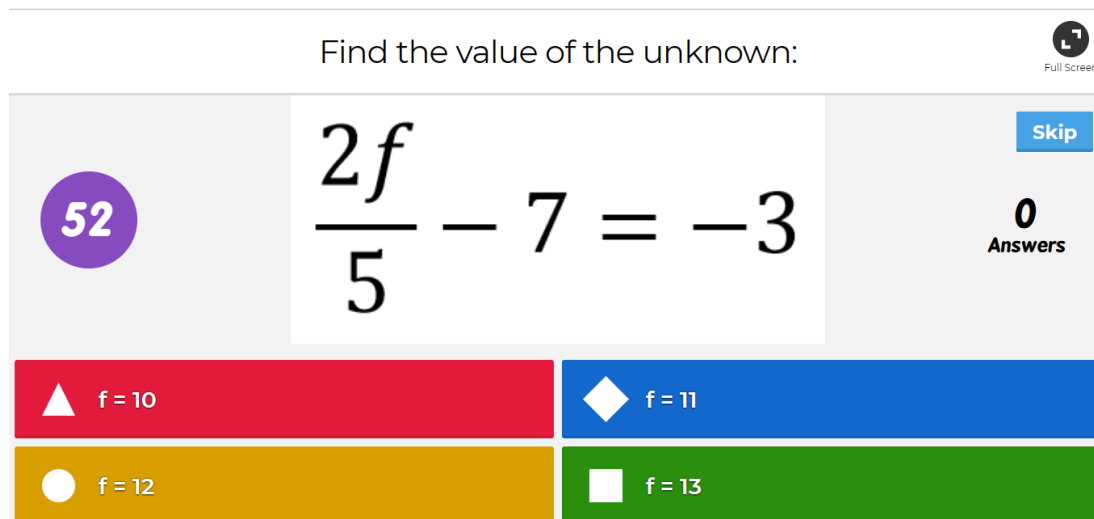


Fig.34. Kahoot. Sample question

After answering all the question, *Kahoot!* Instantly provides an Excel document showing the general overall performance, the individual final scores and a question summary, which is a great tool for self-assessment:

Overall Performance				
Total correct answers (%)			60,15%	
Total incorrect answers (%)			39,85%	
Average score (points)			10593,32 points	
Final Scores				
Rank	Players	Total Score (points)	Correct Answers	Incorrect Answers
1	Daniel	19055	16	2
2	Antonio	18168	15	3
3	Gonzalo	16071	14	4
4	Carmen	15682	14	4
5	Lorena	13739	13	5

Fig.35. Kahoot's report. Overall performance and individual final scores

8.2.8. Session 8: Review of contents with Interactive Groups II

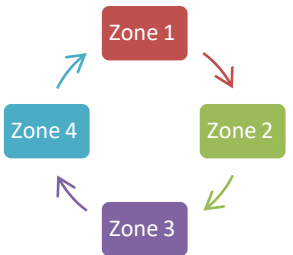
TIME	ACTIVITIES		CONTENT		MATERIALS & RESOURCES
60'	Interactive Groups: 	HOTS	SUBJECT	Resolution of first degree equations with one unknown variable. Interpretation of solutions. Problem solving with equations.	Zones 1, 2, 3 & 4
				Confidence in the ability to develop appropriate attitudes and to deal with the difficulties of scientific work.	
			LANGUAGE	Comprehension of written short texts or expressions.	BICS
				Oral expression of ideas and arguments to support or refute a mathematical procedure or a problem-solving process.	BICS
				A positive attitude to and self-confidence when learning a foreign language.	BICS

Table 10. Session 8

Zone 1: SudoMaths of equations

STEP 1: Students have to solve the following equations and write the solutions in the corresponding cell of the Sudoku template. There are many equations to solve, therefore, they should work as a team and divide up the tasks.

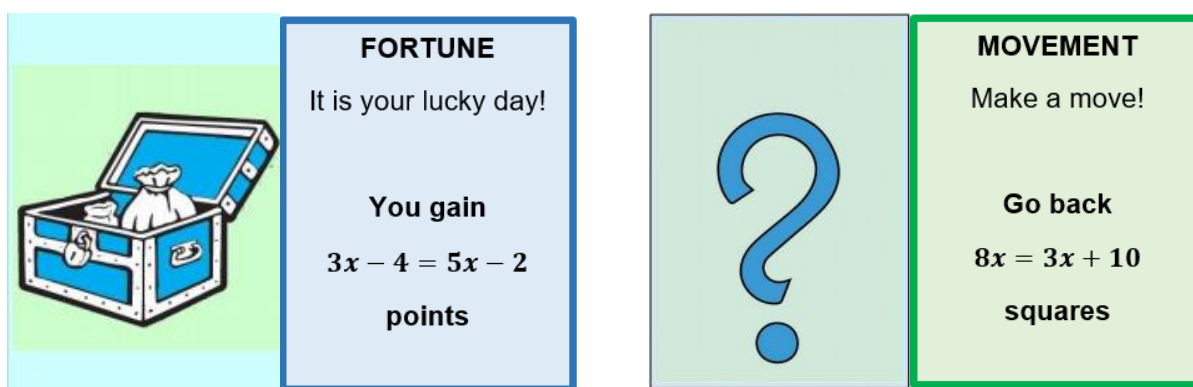
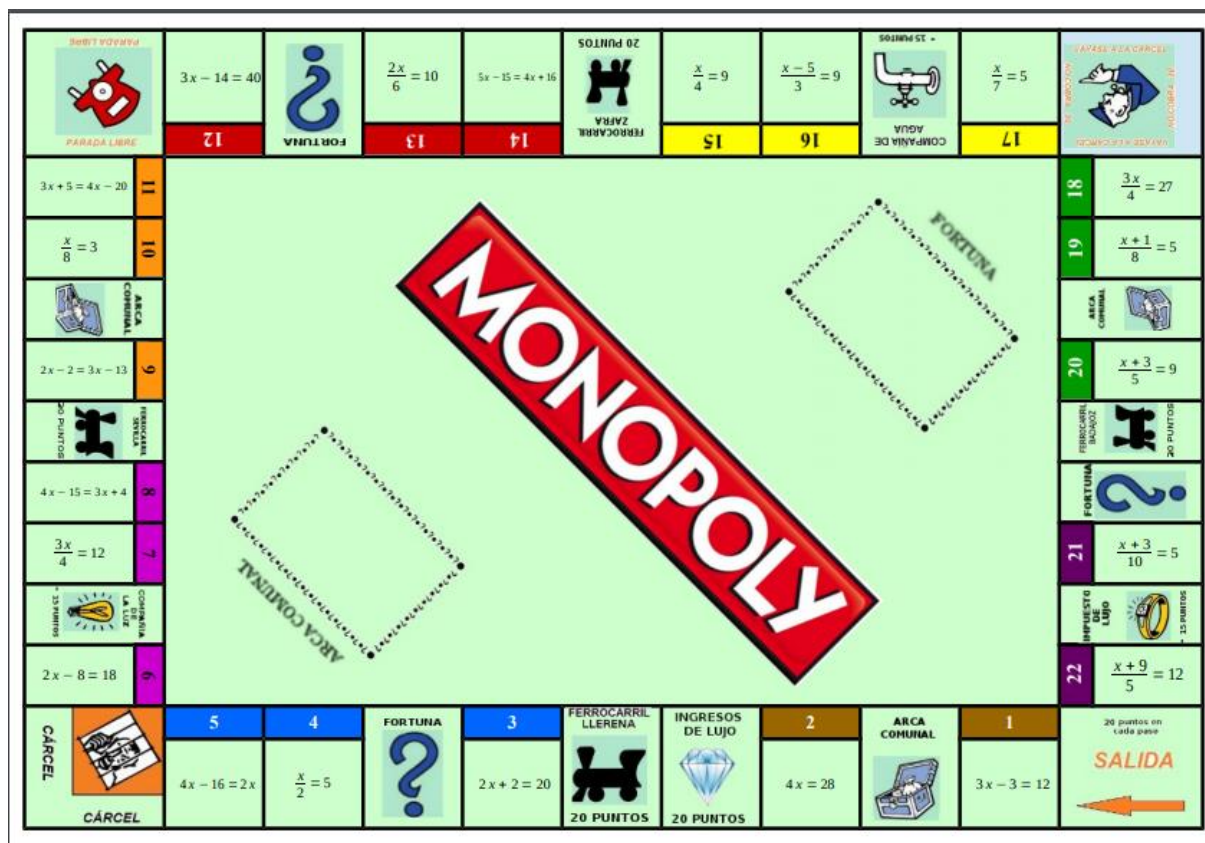
	ECUACIÓN	Sol.		ECUACIÓN	Sol.
A1	$3a+4=a+18$		A2	$6 - 3(2p-4)= -18$	
A3	$2c-3(c-4)=c+2$		A6	$0,5t - 3t+5=0$	
A9	$4(1/4 +x)=5$		B6	$1- 6(y+3) = -23$	
B7	$-2s -5 = -11$		C1	$4u -7 = 5 -2u$	
D3	$-2x-13= -3x-5$		D5	$5z +2= 2z +5$	
D8	$4y+9=-6y -5$		E1	$4s -2s =18$	
E6	$2/3 + 3t/4 = 31/6$		E8	$7x-10=x+2$	
E9	$-9 = p -14$		F4	$2(a+2)=3(a-1)$	
F7	$4/6 = m/9$		F8	$2b +4=6b -32$	
G1	$-4(x+6)= -40$		G3	$2x -7=20-x$	
G4	$8y-(2y-3)=9$		H4	$2(8+p)=22$	
I2	$3d - (d+4)= -2$		I6	$5c -3=2c +12$	
I8	$3x -2 =16$				

STEP 2: Once completed, students must solve the Sudoku following its common rules.

	1	2	3	4	5	6	7	8	9
A									
B									
C									
D									
E									
F									
G									
H									
I									

Zone 2: Monopoly of equations

The activity is based on the classic Monopoly game. Here is the board and a sample of some cards:



The game basically involves solving first degree equations to move forward. For example, students should solve an equation correctly if they want acquire a property. In this game, a volunteer has the answer key and he/she is responsible for ensuring that the group plays correctly.

Zone 3: Pentamino Puzzle

As at the previous session with interactive groups, students are provided with 12 coloured wooden pieces, which are labelled using first degree equations:



Fig.36. Pentomino. Wooden pieces

Students have to put the pieces together in the proper order. This time, students must find a different animal. In this case they should get a kangaroo:

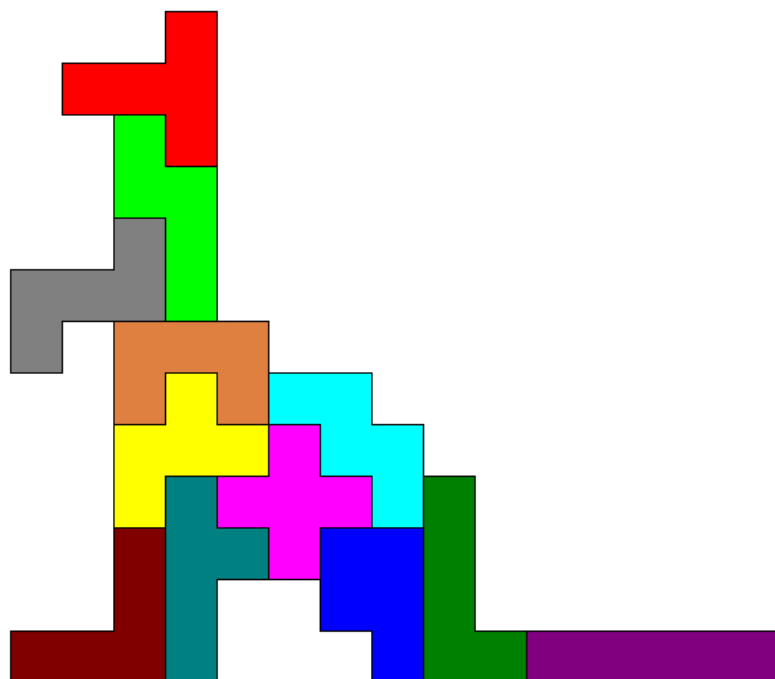
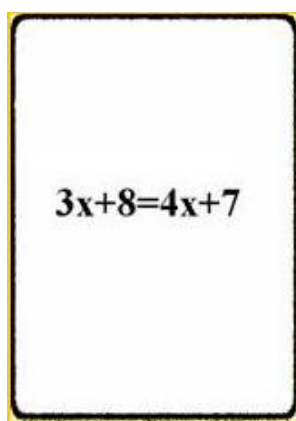


Fig.37. Pentomino. Kangaroo

Zone 4: “Chinchón” of equations

This game is based on the traditional Spanish card game “Chinchón”. There are 5 families of cards: family 1 is formed by cards with equations whose solution is 1, family 2 is formed by cards with equations whose solution is 2, and so on, up to cards with solution 5. For example, the following card belongs to family 1 because its solution is 1:


$$3x+8=4x+7$$

The adult volunteer deals out the cards. The rules are simple: taking turns, students must put the cards in columns and in numerical order according to the family each card belongs to:



Several columns may be formed. However, students lose the turn if they do not have cards to follow the numerical order of the columns. The volunteer is provided with an answer key to make sure of the success of the game.

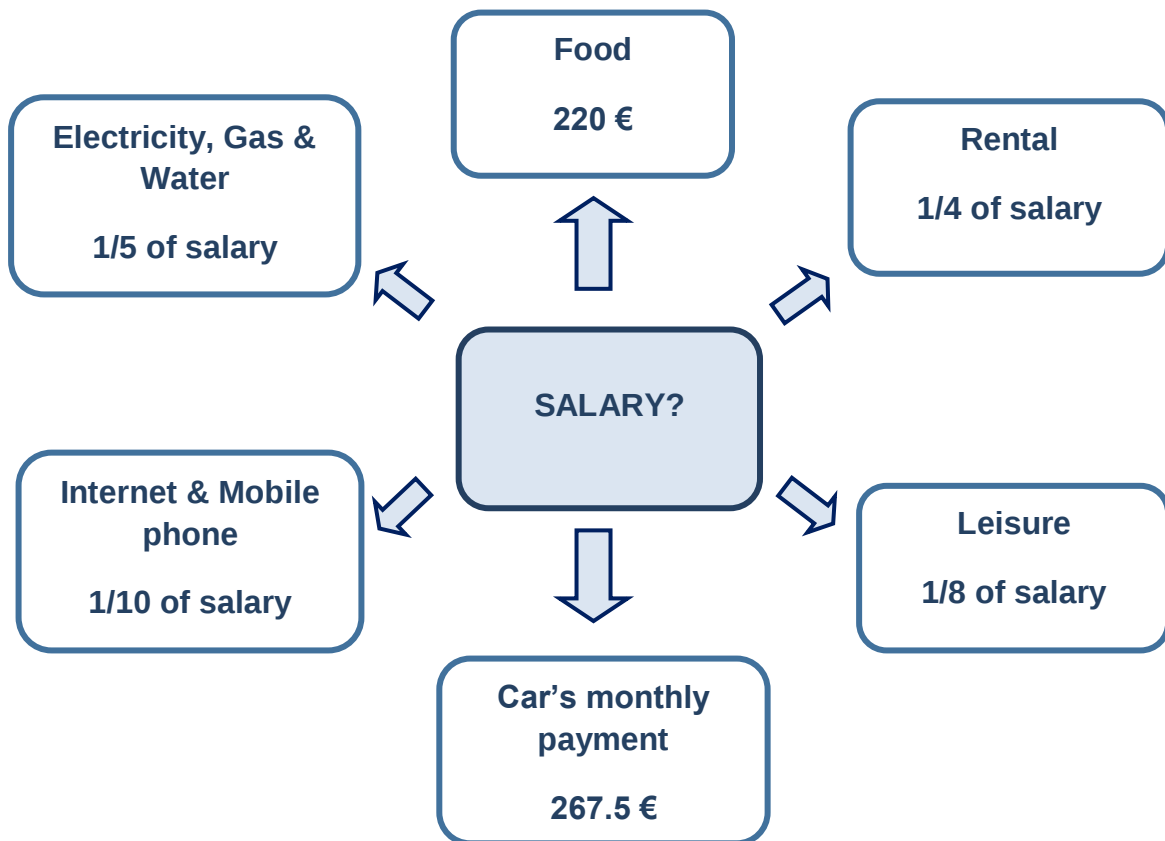
8.2.9. Session 9: Problem-solving II

TIME	ACTIVITIES		CONTENT			MATERIALS & RESOURCES
60'	Work in groups: The class is divided into 4 groups. There are 2 problems to solve, so that each problem will be solved by two different groups in order to compare results. Once the problem has been solved, the groups should make an oral presentation to discuss their findings.	HOTS	SUBJECT	Planning of the resolution process of a problem.		Sample 11 & Sample 12
				Confidence in the ability to develop appropriate attitudes and to deal with the difficulties of scientific work.		
				Resolution of first degree equations with one unknown variable. Interpretation of solutions. Problem solving with equations.		
			LANGUAGE	Comprehension of written short texts or expressions	BICS	
				Oral expression of ideas and arguments to support or refute a mathematical procedure or a problem-solving process.	BICS & CALP	

Table 11. Session 9

Sample 11: Domestic economy II → Income Distribution

A company has offered you a job, but you have to negotiate contract conditions and salary. The scheme below shows the distribution of your monthly costs:



a) What should be the minimum salary that you would have to negotiate with the company in order to **bear the costs**?



Fig.38. Debt&Income. Retrieved May 29, 2019 from <https://www.wsfcu.org/understanding-your-debt-to-income-ratio/>

b) Where would you like to work? Think of a job. Do you think that women and men would have the same **recruitment conditions**? Discuss with your peers.

c) Add the unknown words and expression in your glossary of terms.

Sample 12: Domestic economy II → Opening a savings account

Your grandparents give you money from time to time. You want to place an amount of this money, x , in a **savings account**. The banker tells you that the **interest rate** is r % compounded annually, that is to say, after n years the savings S will be:

$$S = x \cdot \left(1 + \frac{r}{100}\right)^n$$



Fig.39. Savings. Retrieved May 29, 2019 from <https://www.finder.com/savings-account-compound-interest>

- a) What is the interest rate? Discuss with your peers.
- b) If the interest rate is 5% and you place an amount of 250 €, how much money will you have after 1 year?
- c) Find the amount of money x that you would have to place if you want to save 300 € in a year (interest rate = 5%).
- d) Add the unknown words or expression in your glossary of terms.

Comments: In these problems, students work on Mathematics and domestic economy again in order to highlight the numerous applications of Mathematics and its connections with other areas of knowledge. As we have said above, it is essential to include connections between the subject content and other topics to achieve students' motivation and cognitive engagement.

8.2.10. Session 10: Second degree equations I

TIME	ACTIVITIES		CONTENT			MATERIALS & RESOURCES	
30'	Flipped Classroom	HOTS	SUBJECT	Resolution of second degree equations with one unknown variable.		Video available at https://www.youtube.com/watch?v=drcVCRGX40w	
			LANGUAGE	Overcoming insecurities and dealing with unknown situations when speaking orally in a foreign language.	BICS & CALP		
30'	Revision of common errors when solving a second degree equation and proposal of a video recording.	LOTS	SUBJECT	Resolution of second degree equations with one unknown variable.		Sample 13	
				Confidence in the ability to develop appropriate attitudes and to deal with the difficulties of scientific work.			
			LANGUAGE	Comprehension of written short texts or expressions.			BICS & CALP
				A respectful attitude towards people who speak in a foreign language.			BICS

Table 12. Session 10

Sample 13: Revision of common errors and proposal of a video recording

First, the teacher exposes some of the most common errors when working with second degree equations. Here are some of these errors:

Let's consider the equation $x^2 - 3x + 2 = 0$, where $a = 1$, $b = -3$, $c = 2$.

Applying the formula, we get:

$$x = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} = \frac{-3 \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{3 \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} = (*)$$

Mistake 1): The coefficient b is a negative number, so it changes its sign when we apply $-b$ in the formula

$$(*) = \frac{3 \pm \sqrt{-9 - 4 \cdot 1 \cdot 2}}{2} = \frac{3 \pm \sqrt{9 - 4 \cdot 1 \cdot 2}}{2} = \frac{3 \pm \sqrt{5 \cdot 1 \cdot 2}}{2} = \frac{3 \pm \sqrt{9 - 8}}{2} = (*)$$

Mistake 2): You usually write -9 instead of 9. As the negative sign is inside parenthesis, the power affects -3

Mistake 3): Using BIDMAS rule, we do first the product $4 \cdot 1 \cdot 2$ instead of doing $9 - 4 = 5$

$$(*) = \frac{3 \pm \sqrt{1}}{2} = \frac{3 \pm 1}{2} = \begin{cases} \frac{3+1}{2} = \frac{4}{2} = 2 \\ \frac{3-1}{2} = \frac{2}{2} = 1 \end{cases}$$

Then, students are asked to record a short video that shows one of these common errors or a different one. The video will be displayed in the classroom at the end of the unit. To further illustrate this, we can play the following video:

<https://www.youtube.com/watch?v=IB-gXkcZGsY>

Although this video is recorded in Spanish, it shows well the main idea of the task. Naturally, students are requested to record the video in English and they should speak both conversational and academic language.

8.2.11. Session 11: Second degree equations II

TIME	ACTIVITIES		CONTENT			MATERIALS & RESOURCES
35'	Whole class activity: “The Appointments’ Clock” for solving second degree equations.	LOTS	SUBJECT	Resolution of second degree equations with one unknown variable.		Worksheet 6
			LANGUAGE	Oral expression of ideas and arguments to support or refute a mathematical procedure or a problem-solving process.	BICS & CALP	
25'	Work in pairs: “Equations Cryptogram” worksheet to work our second degree equations and discuss about homophobia.	LOTS	SUBJECT	Resolution of second degree equations with one unknown variable.		Worksheet 7
			LANGUAGE	Overcoming insecurities and dealing with unknown situations when speaking orally in a foreign language.	BICS	
				A respectful attitude towards people who speak in a foreign language	BICS	

Table 13. Session 11

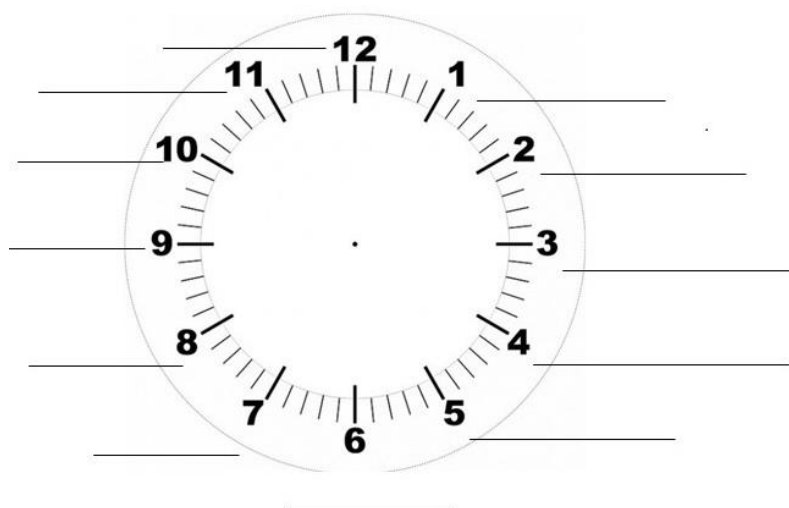
Worksheet 6: The Appointments' Clock

THE APPOINTMENTS' CLOCK

Here is the appointments' clock. You have to **arrange a meeting** with a classmate for each one of the hours. You should arrange each hour with a different peer and write his/her name in the lines below.

When the teacher says a particular hour, you have to **meet with your classmate and solve the associated equation**.

The person with the greatest number of well-solved equations will win a price.



1	$x^2 - 4x + 4 = 0$	7	$x^2 - 6x + 9 = 0$
2	$x^2 - 2x = 0$	8	$x^2 - 49 = 0$
3	$x^2 - 36 = 0$	9	$2x^2 - 6x = 0$
4	$x^2 - 3x + 2 = 0$	10	$x^2 - 2x + 1 = 0$
5	$x^2 + 5x = 0$	11	$4x^2 + 12x = 0$
6	$x^2 - 81 = 0$	12	$x^2 - 1 = 0$

Comments: this activity involves the whole class to increase students' interest and motivation. The teacher moves around the classroom giving support and checking that students talk in English. At the end, a whole-class feedback will give cohesion to the learning process.

Worksheet 7: Equations Cryptogram

CRYPTOGRAM OF EQUATIONS: DISCOVERING THE HIDDEN MESSAGE

Solve the following equations. You will get 1 or 2 solutions. Choose the smaller solution and assign a letter using the code. You will discover a secret message.

P = 4	H = -3	T = -1/3	A = 3	O = 1
S = 5	B = -2	I = 4	M = -1	

$x^2 - 10x + 25 = 0$	S
$9x^2 + 6x + 1 = 0$	T
$x^2 - 2x + 1 = 0$	O
$x^2 - 10x + 21 = 0$	P

$x^2 + 2x - 3 = 0$	H
$x^2 - 3x + 2 = 0$	O
$x^2 - x - 2 = 0$	M
$x^2 - 4x + 3 = 0$	O
$x^2 - 7x + 12 = 0$	P
$5x^2 + 14x - 3 = 0$	H
$x^2 - 5x + 4 = 0$	O
$x^2 - x - 6 = 0$	B
$x^2 - 8x + 16 = 0$	I
$x^2 - 6x + 9 = 0$	A

Discussion:

- a) How many people with blue eyes do you know? Is it usual? Is it natural?
- b) What does the acronym LGTBIQ mean?
- c) Have you ever seen acts of homophobia in the school? What can you do about it?

Comments: in this activity we focus on second degree equations and homophobia, which is a key cross-curricular issue. After solving equations and guessing the hidden message, a whole class debate takes place in the classroom. Students must share their ideas about homophobia and propose measures to eradicate

8.2.12. Session 12: Assessment

TIME	ACTIVITIES	CONTENT	MATERIALS & RESOURCES
20'	Assessment test with Plickers	Students are required to do a final unit test with the application Plickers, which has been used throughout the unit. Students do the test individually, and their responses are corrected by the application at the time when they hold up their cards. Hence, they receive immediate feedback and a final mark of the test.	Questions are similar to those carried out in sessions 3 and 7 (See Samples 3 and 8)
30'	Self-Assessment of the unit	Students are asked to complete a self-assessment template to rate their level of understanding of the unit. The goal is simple: students must be aware of their achievements and their performance. Moreover, the teacher should check their self-assessment templates, and could also ask questions about any aspect of the didactic unit to make sure their answers are in line with their real performance.	Sample 14
10'	Peer-Assessment of Interactive groups	The objective of this type of assessment is to give students a pause to reflect on interactive groups' sessions. They are asked to evaluate some aspects of the team work when working with interactive groups' methodology, as well as to assess their peers.	Sample 15

Table 14. Session 12

Sample 14: Self-Assessment of the unit²

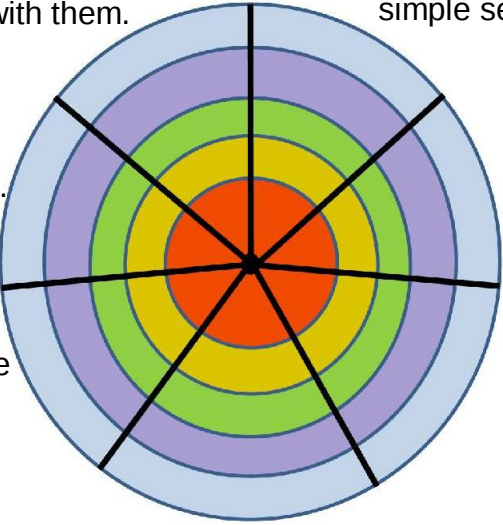
SELF-ASSESSMENT ALGEBRA

NAME: _____

1. I can translate everyday expressions into algebraic language.

2. I can simplify algebraic expressions and do basic operations with them.

7. I can work in team and distribute tasks and responsibilities in a respectful way.



3. I can solve first-degree equations and simple second-degree equations.

4. I can comprehend word-problems and use algebra as a tool to solve them.

5. I can express orally ideas and arguments to support or refute a mathematical procedure or a problem-solving process.

6. I can connect new concepts with real situations, personal experiences and past learning.

1

2

3

4

5

I'm not sure

I know it perfectly

² Idea and image taken from Bueno, A. and Plata, C. 2018. *Unit 3: Strategies and tools for assessment*. Universidades de Jaén y Córdoba: Máster Interuniversitario en Enseñanza Bilingüe y Aprendizaje Integrado de Contenidos y Lenguas Extranjeras.

Sample 15: Peer-Assessment in interactive groups

GROUP WORK				
	1	2	3	4
Group contribution	The group is rarely focused on the task and its members do not contribute to the overall goal of the group at all.	The group is sometimes focused on the task and some members occasionally contribute to the overall goal of the group.	The group is focused on the task most of the time and everyone contributes to the overall goal of the group.	The group is always focused on the task and everyone contributes to the overall goal of the group.
Share tasks and help others	The group needs reminders to divide tasks among all the members and anyone offers help.	Tasks are not equally divided among all the members of the group and not everyone is willing to help.	Tasks are divided equally among all the members of the group but not everyone is willing to help.	Tasks are divided equally among all the members of the group and everyone is always willing to help.
Respect	Nobody listens, interacts and discusses in a respectful manner and some members don't show understanding for the learning needs of others.	Some members don't listen, interact and discuss in a respectful manner and don't show understanding for the learning needs of others.	Everyone always listens, interacts and discusses in a respectful manner but some members don't show understanding for the learning needs of others.	Everyone always listens, interacts and discusses in a respectful manner. Everyone shows understanding for the learning needs of others.
INDIVIDUAL WORK				
NAME:	1: Never	2: Sometimes	3: Often	4: Always
	Gives ideas	Takes responsibility	Helps others	Respects others

Table 15. Rubric for peer-assessment in interactive groups

9. END-OF-UNIT ASSESSMENT

As is the case with learning, assessment theory and practice have also been evolving over the years (Cumming & Maxwell 1999). Assessment based exclusively on traditional performance tests has been declining owing to the fact that tests are not very useful to guide learners on their on-going learning process (Black & Wiliam 2003). This section addresses the assessment of the didactic unit as a whole.

First, it is necessary to clarify the term “assessment” in the CLIL context, which can be defined in an accurate and complete way as:

Assessment [...] is the achievement of content and language goals (planned outcomes); achievement of learning skills goals; use of language for various purposes (i.e., academic, social, business registers); ability to work with authentic materials, as well as with native and non-native speakers of the CLIL language; willingness to experiment with content and language; ongoing growth of language (avoiding ‘plateauing’).

(Mehisto 2012: 20)

Hence, assessment focuses on how students perform in a wide range of areas and skills, so the term is linked with the well-known key competences in the sense that the main goal is to assess the students’ capacity to apply the learning in order to solve problems in a given situation and context. In other words, we may say that the term “assessment” has a **competency-based learning approach**.

Traditionally, in Spain -and particularly in Andalusia- assessment is understood as the measurement of what students have learned: what they know, what they can do and how they do it. Teachers are required to give a final mark at the end of each school term as well as at the end of the school year and, as we know, CLIL has a dual focus – subject and language – therefore it is logical to assume that both subject and language content should be assessed. Nevertheless, although it seems contradictory, language assessment is ill-defined in laws and in official regulations, in the sense that assessment criteria for language content are not defined. In fact, the

Order of 28th June 2008, which regulates bilingual education in Andalusia, states in Article 8:

En la evaluación de las áreas, materias o módulos profesionales no lingüísticos primarán los currículos propios del área, materia o módulo profesional sobre las producciones lingüísticas en la L2. Las competencias lingüísticas alcanzadas por el alumnado en la L2 serán tenidas en cuenta en la evaluación del área, materia o módulo profesional no lingüístico, en su caso, para mejorar los resultados obtenidos por el alumnado, de acuerdo con los criterios de evaluación definidos en el proyecto educativo.

(BOJA 2011: 7)

Moreover, the official **Instructions of 7th June 2018**, which add up some details to the previous Order, in its 7th instruction establishes:

El profesorado de lengua extranjera será el responsable de evaluar la competencia lingüística del alumnado [...] El profesorado de ANL y MPNL tendrá en cuenta en su evaluación los descriptores del nivel de competencia lingüística alcanzado por el alumnado de acuerdo con el MCERL, si bien priorizará el desarrollo de los objetivos propios del área, materia o módulo profesional sobre la producción lingüística, que no deberá influir negativamente en la valoración final del área.

(Junta de Andalucía 2018: 7)

Consequently, it is not up to bilingual teachers to assess language content because they are not entitled to by law. Language content, therefore, must be assessed by language teachers. Furthermore, as the afore-mentioned instruction states, we need to be mindful of the fact that subject content is the priority both in teaching and in assessment.

Under these circumstances, it can be said that in CLIL the foreign language is the means by which students achieve the subject content, but it is not the ultimate goal in assessment. That is why in this didactic unit we assess only the **evaluation criteria defined in the Order of 14th July 2016**. Accordingly, the official **evaluation criteria covered in this didactic unit** are:

- **E.C.1.1.** To express orally and reasonably the process followed in the resolution of a word-problem.
- **E.C.1.9.** To overcome mental blocks and insecurities in unknown situations.
- **E.C.1.12.** To use information and communication technologies in the learning process to search, analyse and collect information from the Internet or other resources in order to produce documents, to make presentations and to share them and facilitate interaction.
- **E.C.2.6.** To analyse changing numerical processes, to identify patterns and use algebraic language in order to express them and operate with them.
- **E.C.2.7.** To use algebraic language to symbolise and to solve problems by means of equations and applying both algebraic and graphic methods to check the obtained results

As can be noted from the above criteria, none of them are directly associated with language content but rather with Mathematics. However, we may say that we can indirectly relate some of them to the foreign language. After all, subject and language are developed at the same time through the CLIL activities proposed, and thus they can be assessed at once. For instance, evaluation criteria E.C.1.1. alludes to oral expression of a problem resolution process, that is to say, when we assessing E.C.1.1. we are indirectly assessing oral language skills.

Below is a table showing a summary of how we assess the evaluation criteria that this didactic unit covers. It puts together the assessment tools, the contexts of assessment and the evaluation criteria in order to judge the learning process. Moreover, it should be noted that it is not intended to be an end-of-course assessment but rather a summative one in the middle of the academic year.

			PERFORMANCE LEVEL			
EVALUATION CRITERIA	TOOLS	CONTEXTS	1 Unsatisfactory	2 Emerging	3 Proficient	4 Exemplary
E.C.1.1.	Rubric	Oral presentations	Speaks very little or doesn't speak at all.	Speaks with some hesitation and attempts to construct a mathematical argument.	Speaks fluently but arguments are not enough reasoned.	Speaks fluently and arguments are constructed with adequate mathematical basis.
		Interactive groups				
	Teacher Logbook	Debates				
E.C.1.9.	Rubric	Oral presentations	Refuses to deal with unknown situations.	Needs help to overcome mental blocks when dealing with unknown situations.	Quickly recovers from mental blocks when dealing with unknown situations.	Controls mental blocks very well when dealing with unknown situations.
	Teacher Logbook	Debates				
		Problem-solving				
E.C.1.12.	Teacher Logbook	ICT sessions	Has difficulties to operate the ICT and to identify information on it.	Operates properly the ICT but has difficulties to identify information on it.	Operates successfully the ICT but doesn't get the most from it.	Operates successfully the ICT and make it a valuable tool for learning.
	Rubric	Video recording				

E.C.2.6.	Peer-assessment rubric	Notebook	Doesn't translate expressions into algebraic language or can't do algebraic operations.	Translates expressions into algebraic language but makes many errors when handling algebraic operations.	Translates expressions into algebraic language and makes minor errors when handling algebraic operations.	Translates expressions into algebraic language and successfully handles algebraic operations.
	Teacher Logbook	Worksheets				
	Rubric	Interactive groups				
	Plickers	Final unit test				
E.C.2.7.	Peer-assessment rubric	Notebook	Doesn't choose any strategy or chooses a strategy that will not lead to a solution of the problem.	Chooses a partially correct strategy or choose a correct strategy for only one part of the problem.	Chooses a correct strategy and applies it to the problem-solving situation.	Plans in advance, chooses an efficient strategy and applies it to the problem-solving situation. Checks the result.
	Teacher Logbook	Worksheets				
	Rubric	Interactive groups				
	Plickers	Final unit test				

Table 16. Summary of end-of-unit assessment

In this didactic unit, students work on the contents defined in the 4Cs framework table (table 2), and each one can be easily linked with one evaluation criteria due to the similarity between the writing of the official contents and the official evaluation criteria. The didactic unit includes 12 sessions (the last one is dedicated to assessment), and in each session students work two or three contents, it means that we focus on only two or three evaluation criteria in a session. Below there is a summary table that links the contents studied and the evaluation criteria covered in this didactic unit:

SUBJECT & LANGUAGE CONTENTS 4C'S FRAMEWORK (Table 1)		EVALUATION CRITERIA ASSOCIATED				
		E.C.1.1.	E.C.1.9.	E.C.1.12.	E.C.2.6.	E.C.2.7.
Content	CT1				X	
	CT2				X	
	CT3					X
	CT4					X
	CT5				X	
	CT6					X
Cognition	CG1	X				
	CG2					X
	CG3				X	
	CG4				X	
	CG5		X			
Communication	CM1	X				
	CM2		X			
Culture	CU1			X		
	CU2		X			
	CU3		X			
	CU4		X			

Table 17. Connection between contents and evaluation criteria

To assess the whole didactic unit, a truly fair option would be give more weight to evaluation criteria that are appear more times in the didactic unit. The reason is clear: if an evaluation criteria appears more times, students will work more on the

associated subject and language content and, therefore, this criteria has more importance in the unit. After counting the times that each content appears in the didactic unit, and taking into account the connections in the above table, the end-of-unit assessment reads as follows:

Evaluation criteria	Number of times that it appears in the didactic unit	Weight (%)
E.C.1.1.	16	16 / 66 → 24%
E.C.1.9.	20	20 / 66 → 30%
E.C.1.12.	3	3 / 66 → 5%
E.C.2.6.	13	13 / 66 → 20%
E.C.2.7.	14	14 / 66 → 21%
Total	66	66 / 66 → 100%

Table 18. Weight of the evaluation criteria covered in the didactic unit

As we can see, evaluation criteria linked with the subject content (2.6. and 2.7.) represent a 41% of the final mark, while the rest have a weight of 59%. Therefore, assessment gives more importance to evaluation criteria related with attitudinal skills, which are essential in the first levels of Compulsory Secondary Education.

10. CONCLUSIONS

The aim of the present dissertation has been to develop a full didactic unit for the subject of Mathematics and for students of 2nd year of Compulsory Secondary Education.

There are many challenges in implementing a CLIL didactic unit: one of the most significant is methodology. There is not a single correct way of teaching in CLIL. In my opinion, the key to success in CLIL is include a wide range of models which may change over time depending on learning needs. CLIL methodologies are a mode of operation which involve innovation, collaborative work, student-centred approaches, scaffolding, ICT, problem-solving, authentic materials and much more. Regarding the foreign language, I consider that the main thing is to use it as a means of

communication and as a vehicle to learn content. It goes without saying that CLIL is not just a text book written in English. In fact, being part of a bilingual program is not about speaking two languages but developing communicative skills and to provide a certain “mental flexibility”. Focusing CLIL methodologies in this way gives the students the advantage of developing a higher level of mental concentration because they have to work with a content in a different language from their mother tongue. Ultimately, activities and methodologies used in a didactic unit must be designed so that students have an active role and they reach the target language level almost without realising.

Another challenge in a CLIL didactic unit is assessment. Although laws do not prescribe language assessment, I believe that it is essential, otherwise the dual approach content-language of CLIL becomes worthless. So we need well-defined government guidelines in order to standardize criteria. Nevertheless, some difficulties may emerge when assessing. For example, if a student performs unsatisfactorily, is it due to his/her poor understanding of concepts? or because he/she cannot express them orally in a foreign language? Or maybe the student does not comprehend the question? The challenge is to assess subject and language at the same time and through the same tasks and activities, although priority would be given to assessment of the subject content.

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